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J. R. HIND, Esq., F.R.S., President, in the Chair.

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Lieut. Sidney Smith, R.N., H.M.S. *Forester*;

Capt. Robert Wright Sterry, 82 Basinghall Street, E.C.; and

Capt. Peter Thompson, Marine Board, Tower Hill, E.C.;

were balloted for and duly elected Fellows of the Society.

On the Determination of the Solar Parallax by Means of the Parallactic Inequality in the Motion of the Moon. By James Campbell, Esq., and E. Neison, Esq. (Conclusion of the Paper in the May No., pp. 386–411.)

The previous portion of this communication has been devoted to the consideration of the general features of the problem of determining the value of the coefficient of the parallactic inequality, and to the critical examination of the results which have been obtained by previous investigators. The present portion contains a new discussion of the Greenwich Meridian observations of the Moon made during the fifteen years between 1862 and 1876. During this period the Greenwich observations have been systematically reduced and compared with the tabular places of the Moon derived from Hansen's *Tables de la Lune*, and

they form a mass of data far superior to any which have hitherto been employed for the determination of the value of the coefficient of the parallactic inequality.

As a first approximation these observations will be reduced in the manner employed by Mr. Stone and Prof. Newcomb for determining the value of this coefficient, and this step will serve to show the real nature of the assumptions made by them as the basis of their investigations.

Let

δL = the error in the mean longitude of the Moon at a fixed epoch,

E = the error in the tabular longitude of the Moon found by subtracting the observed place from the place assigned by Hansen's tables.

In accordance with the assumption made by Mr. Stone and Prof. Newcomb, that the errors in the tables might be regarded as due to errors in the mean longitude and errors in the parallactic inequality, every observation will furnish an equation of the form

$$\delta L + \delta P \sin \theta = E.$$

Then, in conformity with the same method, every observation was selected for which the value of $\sin \theta$ exceeded 0·90, this being equivalent to taking observations within two days of the first and third Quarters of the Moon. The observations were divided into yearly groups, each group commencing in the October of one year and ending in the October of the next, this division being adopted so as to eliminate as far as possible any discordances due to errors in the annual equation. For each group δL would represent the error of the mean longitude of the Moon on March 31st.

First, all the observations for which $\sin \theta$ was positive were added together and the mean taken. If e' denotes the mean of the errors of the tables and a' the mean of the values of $\sin \theta$, there will result by this means an expression of the form

$$\delta L + a' \delta P = e'.$$

Treating the observations for which $\sin \theta$ was negative in the same manner, and putting e''' and a''' for the corresponding values denoted by e' and a' at the first Quarter, an expression is obtained of the form

$$\delta L - a''' \delta P = e'''.$$

Therefore, eliminating δL, and dividing by the coefficient of δP.

$$\delta P = \frac{e' - e'''}{a' + a'''}.$$

As no correction has been introduced for the variation in the semi-diameter, this result will be really $\delta(P)$, the correction to the apparent coefficient.

In this manner a value for $\delta(P)$ was obtained for each yearly group. These values are

Year.	Number of Observations.	Value of $\delta(P)$.	Difference from the Mean.
1876–1875	35	$\delta(P) = -2''.39$	$-0''.71$
1875–1874	35	$= -1''.26$	$+0''.42$
1874–1873	52	$= -0''.89$	$+0''.79$
1873–1872	31	$= -1''.00$	$+0''.68$
1872–1871	40	$= -1''.66$	$+0''.02$
1871–1870	38	$= -1''.66$	$+0''.02$
1870–1869	31	$= -1''.97$	$-0''.29$
1869–1868	35	$= -1''.42$	$+0''.26$
1868–1867	47	$= -2''.26$	$-0''.58$
1867–1866	38	$= -1''.81$	$-0''.13$
1866–1865	36	$= -1''.51$	$+0''.17$
1865–1864	37	$= -2''.32$	$-0''.64$
1864–1863	38	$= -1''.98$	$-0''.30$
1863–1862	34	$= -1''.64$	$+0''.04$

These results are moderately accordant, the greatest difference from the mean being less than four-fifths of a second, or less than a moiety of the correction, and the mean of all the discordances is only one-third of a second. Dividing the observations into two seven-year groups, in order to detect any systematic variation, the results are:—

From 265 observations during the period 1862–1869

$$\delta(P) = -1''.85 \pm 0''.091.$$

From 262 observations during the period 1869–1876

$$\delta(P) = -1''.50 \pm 0''.138.$$

These results require to be corrected for any error in the value adopted by Hansen for the horizontal semi-diameter of the Moon. This must be determined from the observation of the transits of both limbs made when the Moon is Full. During the period 1862–1877 there have been 38 such observations made, but five of these were observed by computers only occasionally engaged in observation, and are liable to unknown errors. These five must therefore be rejected, and only those observations employed which were made by the astronomers who systematically observe the Moon. Put δS for the excess of the tabular over the observed semi-diameter; then from 22 accordant observations during the period 1862–1869

$$\delta S = +0''.37 \pm 0''.10.$$

Next, from 11 accordant observations during the period 1869–1877

$$\delta S = +0''.10 \pm 0''.08.$$

These corrections are to be applied as they stand to the values obtained above for $\delta(P)$, or the apparent value of the parallactic inequality by the formula

$$(P)+\delta(P)+\delta S = P_1 = -126''.46.$$

Accordingly we have:—For the group 1862–1869

$$(P) = -124''.98 \pm 0''.14.$$

For the group 1869–1876

$$(P) = -125''.08 \pm 0''.16.$$

The two results are beautifully accordant.

Taking, therefore, the entire group in one, we have from 527 observations during 1862–1876

$$(P) = -125''.06 \pm 0''.10.$$

This result does not seem to harmonise with the value found by Prof. Newcomb, which was

$$(P) = -124''.41;$$

but it must be remembered that this result obtained by Prof. Newcomb was founded on the observations made during the years 1862–1865 only, and without applying any correction to Hansen's semi-diameter. From the same data the results obtained above would yield the value

$$(P) = -124''.48,$$

in perfect accord with Prof. Newcomb.

On comparing the above result with that deduced from the observations given by Sir G. B. Airy, which on the hypothesis of a forty-five-year inequality is

$$(P) = -124''.11,$$

it will be seen that they differ by $0''.95$. But if this inequality really exists, and does not arise from errors in Airy's tables which do not exist in Hansen's tables, the value found above will require correction for the effect of this inequality. The true value will be therefore

$$(P) = -125''.06+0.80B.$$

If for B be substituted its approximate value $B=1''.20$, we obtain the result

$$(P) = -124''.00,$$

a most remarkable result.

In what precedes we have been engaged in determining the value of the apparent coefficient, (P), but the end in view is really the determination of the true coefficient, P, so as to be able to determine the value of the solar parallax. For this purpose it is necessary to take some steps to determine the difference between the two, arising from the variation in the apparent semi-diameter of the Moon due to difference in the amount of contrast between the lunar limb and the sky.

Let

δI = the difference in the irradiation of the limb of the Full Moon and at its Quarters when seen in bright twilight.

Then the differences in the amount of irradiation at the Moon's limb under different amount of contrast may be expressed by supposing δI to be multiplied by a variable coefficient, i, the value of i being zero for the Full Moon, and gradually increasing up to 1·0 for the time of the Quarter, when the Moon is observed in bright twilight, and up to 1·5 when the Moon is observed in bright daylight. This coefficient i will therefore be a function of the Moon's age or time it transits, and the time of year or brightness of the sky at any hour of the day. The variation in this coefficient is necessarily a little arbitrary, but this is not very important where, as in the present case, the value of δI is to be determined from the observations themselves. The rate of variation which was actually assumed cannot differ much from that assumed by Prof. Newcomb in his memoir on the "Corrections to Hansen's Table," Washington, 1876.

Every observation of the Moon now furnished an equation of the form

$$\delta L + i\delta I + \delta P \sin \theta = E.$$

These equations were now treated exactly as before, but each yearly group was now divided into two portions, a Winter and a Summer portion. In the former the observations were nearly all made on a dark sky, when i was either zero or very small, so that the amount of irradiation was nearly constantly at its maximum. In the latter, the observations were mostly made on a bright twilight sky, and many in broad daylight, when the irradiation was much reduced. By comparing the values found for the parallactic inequality under these different conditions, it would be possible to determine the effect of the change in semi-diameter due to the alteration in the amount of irradiation. By grouping the observations from October to March together, the effect of any error in the annual equation would be materially diminished, as it would be negative from October to December, and positive from January to March, and so disappear in the sum of the two. The same elimination of any error in the annual equation would occur in the summer portion, April to October.

It was found that the mean value of i might approximately be taken as 0·75 in the summer group, and 0·25 in the winter group. These values were adopted.

The results obtained are as follows:—

Summer II. Division.

Group.	No. of Observations.	Resulting Value.	Difference from Mean = −2″·15
1876	21	0·75 δI + δP = −0″·03	+2″·12
1875	22	= −1·79	+ ·36
1874	31	= −2·81	− ·66
1873	17	= −1·98	+ ·17
1872	17	= −3·32	−1·17
1871	16	= −3·05	− ·90
1870	16	= −2·08	+ ·07

Summer I. Division.

			Mean = −1″·84
1869	15	0·75 δI + δP = −0″·52	+1″·32
1868	29	= −1·56	+ ·28
1867	18	= −2·72	− ·88
1866	18	= −2·08	− ·24
1865	16	= −2·72	− ·88
1864	20	= −2·25	− ·41
1863	13	= −0·99	+ ·85

Winter II. Division.

			Mean = −0″·93
1876–1875	14	0·25 δI + δP = −4′·72	−3″·79
1875–1874	13	= −0·73	+ ·20
1874–1873	21	= +1·02	+1·95
1873–1872	14	= −0·01	+ ·92
1872–1871	23	= +0·05	+ ·98
1871–1870	22	= −0·23	+ ·70
1870–1869	15	= −1·86	− ·93

Winter I. Division.

			Mean = −1″·87
1869–1868	20	0·25 δI + δP = −2″·35	− ″·48
1868–1867	18	= −3·02	−1·15
1867–1866	20	= −0·90	+ ·97
1866–1865	18	= −0·95	+ ·92
1865–1864	21	= −1·93	− ·06
1864–1863	18	= −1·72	+ ·15
1863–1862	21	= −2·25	− ·38

Treating the deviations as due to accidental errors of observation, these results yield the following values :—

Summer Division II. 140 observations $0{\cdot}75\,\delta I + \delta P = -2''{\cdot}15 \pm 0''{\cdot}28$,
Summer Division I. 129 observations $0{\cdot}75\,\delta I + \delta P = -1''{\cdot}84 \pm 0''{\cdot}21$,

or uniting the two

Summer 269 observations $0{\cdot}75\,\delta I + \delta P = -2''{\cdot}00 \pm 0''{\cdot}17$.

Next,

Winter Division II. 122 observations $0{\cdot}25\,\delta I + \delta P = -0''{\cdot}93 \pm 0''{\cdot}47$,
Winter Division I. 136 observations $0{\cdot}25\,\delta I + \delta P = -1''{\cdot}87 \pm 0''{\cdot}16$,

or uniting the two

Winter 258 observations $0{\cdot}25\,\delta I + \delta P = -1''{\cdot}40 \pm 0''{\cdot}26$.

Considering the total groups first, by subtracting the winter results from the summer results we have

$$+0{\cdot}50\,\delta I = -0''{\cdot}60 \pm 0''{\cdot}32,$$

or

$$\delta I = -1''{\cdot}20 \pm 0''{\cdot}64.$$

That is to say, the Moon's semi-diameter is decreased in bright twilight by $1''{\cdot}2$ from its value at Full Moon, when it is seen against a black sky. Using this value, we obtain for the value of the true coefficient of the parallactic inequality,

$$P = (P) + 0{\cdot}50\,\delta I,$$
$$= -125''{\cdot}06 - 0''{\cdot}60$$
$$= -125{\cdot}66 \pm 0{\cdot}34.$$

It is obvious, however, that the large probable error for the value found for δI, renders this result most unsatisfactory. Considering the two divisions into which the summer and winter results are each divided, we obtain the values

For the II. Divisions $\delta I = -2''{\cdot}44 \pm 1''{\cdot}10$,
For the I. Divisions $\delta I = +0{\cdot}06 \pm 0{\cdot}54$.

It is evident that there is a systematic difference arising from some cause which is not apparent.

The uncertainty of the value found for δI is best seen by eliminating in the preceding manner the value of δP from each yearly group, and determining the value of δI from the resulting equation. The results are

Year.	Value of δI.
1876–1875	$\delta I = +11''.92$
1875–1874	$= -2''.76$
1874–1873	$= -7''.96$
1873–1872	$= -3''.61$
1872–1871	$= -5''.22$
1871–1870	$= -4''.75$
1870–1869	$= -0''.56$
1869–1868	$= +4''.35$
1868–1867	$= +5''.06$
1867–1866	$= -4''.03$
1866–1865	$= -1''.88$
1865–1864	$= -1''.68$
1864–1863	$= -0''.74$
1863–1862	$= -2''.22$

These results are extremely discordant; neglecting the first value, we obtain from the rest the result

$$\delta I = -1''.66 \pm 0''.69.$$

Including all, the result is

$$\delta I = -0''.68 \pm 0''.92.$$

It is obvious that any results based on such wild values must be fallacious. It shows the misleading character of values obtained from the means of large numbers of observations without examining the separate results to see if they are really accordant.

The result of the preceding investigation might be regarded as discouraging and as showing that it would not be practicable to determine in this manner the effect of the variation in the amount of irradiation at the limb of the Moon. If this were really the case, it would be most unsatisfactory and throw grave doubt on the possibility of determining the true value of the parallactic inequality from observations of the transit of the lunar limb. For the only permissible method of ascertaining the amount of this variation in the irradiation is by this process of comparing the summer with the winter observations. If this prove to be impracticable, it involves as a necessary consequence the failure of the whole method of determining the solar parallax from these observations.

Further consideration shows that these results are too discordant for this view to be adopted. These extreme discordances can only arise from systematic errors due to imperfections in the tables, or method of reducing the observations. These discordances must be due to the failure of the assumption, which

we have adopted, in common with Mr. Stone and Prof. Newcomb, and taken as the basis of our work—the assumption that in the mean of a number of observations all errors except those of the parallactic inequality can be neglected and be trusted to destroy one another.

To determine a more trustworthy value of the coefficient of the parallactic inequality a more elaborate investigation must be made. We proceed, therefore, to this.

Let Hansen's tables be assumed to be affected by the following errors :—

δL = an error in the mean longitude at a given epoch,

δl = an error in the motion of the mean longitude,

δP = an error in the coefficient of the parallactic inequality,

δV = an error in the coefficient of the variation,

δM = an error in the coefficient of the annual equation,

δA = an error in the coefficient of the term with the argument $4\theta - 2\beta$.

Further, suppose Hansen's tables imperfect from not containing terms of the following nature :—

$\delta K \sin k$ = a term with the argument $k = \alpha + 2A - 2J$,

$\delta F \sin f$ = a term with the argument $f = 2\theta - \alpha + \mu - N$,

$\delta F' \sin f'$ = a term with the argument $f' = \alpha - \mu + N$,

where

A = mean longitude of perigee of the lunar orbit,

J = mean longitude of Jupiter,

N = an argument which increases only a few degrees per annum.

Lastly put

$i\delta I$ = the variation in the amount of irradiation at the limb of the Moon,

$t\delta T$ = the effect of the systematic errors in the Greenwich clock star places,

$s\delta S$ = the apparent error of Hansen's value of the horizontal semi-diameter of the Moon,

and

E = apparent error of Hansen's tables.

Then every observation will give an equation of the form

$$\delta L + l\delta l + \delta P \sin p + \delta V \sin v + \delta M \sin \mu + \delta A \sin \alpha + \delta K \sin k + \delta F \sin f + \delta F' \sin f' + i\delta I + t\delta T + s\delta S = E,$$

a system of equations with twelve unknown quantities, or rather variable corrections.

Although in this manner the labour required by the investigation is enormously increased, it has the advantage of furnishing results in which some confidence may be felt, and derived by a process having some pretence to refinement and precision.

The term

$$\delta A \sin \alpha = \delta A \sin (4\theta - 2\beta)$$

introduces a correction necessitated by an error contained in Hansen's tables owing to which the term

$$+0''.335 \sin (4\theta - 2\beta)$$

has been employed instead of the correct term

$$-0''.285 \sin (4\theta - 2\beta).$$

It follows, therefore, that every one of the tabular places given by Hansen's tables requires the correction

$$-0''.62 \sin (4\theta - 2\beta).$$

The necessity for this correction admits of no dispute, and was pointed out by Hansen in a note in his *Darlegung*. Its actual value has been applied to every observation with the effect of markedly improving the accord between the separate observations. As far as the results in the present investigation are concerned, the effect of this correction proves to be trifling.

For the moment passing over the new terms to be introduced, it will be well to consider the effect of the other three corrections.

The first of these is that denoted by

$$i\delta I,$$

which allows for the variation in the amount of irradiation at the limb. The manner in which the coefficient i of this term is supposed to vary has been already explained. The term has been introduced as an indeterminate correction to every observation.

The second of these corrections is that denoted by

$$i\delta T,$$

and arising from the systematic errors in the places of the clock stars used at Greenwich in determining the place of the Moon. The method of applying this correction is as follows. In the Introduction to the Greenwich Nine Year Catalogue is to be found the list of the corrections required by the Right Ascensions of the Greenwich clock stars. From the different volumes of Greenwich Observations it would be possible to ascertain the stars from which the assumed clock-error was derived which was used in each observation, and thus ascertain the exact correction necessary. This would be a work of enormous labour to gain a very small result, for it is only the systematic effect of these errors which need be considered. A sufficiently accurate result was obtained, therefore, by the following method. An examination was made of the actual stars employed for each

observation of the Moon made in the years 1864, 1870, 1877, and it was assumed that the method employed in these years would be also the method employed in the intervening years. It was found that the clock-error used in determining the place of the Moon was derived as a rule mainly from stars which transit within two or three hours of the time of the transit of the Moon. In fact, it was found that the mean place of the Moon for a lunation would be affected by an error closely approximating to the mean error of the clock stars transiting within four hours on either side of ten o'clock. In the summer the stars are within closer and in the winter within rather wider limits than these. Then from the known systematic errors of the clock stars transiting within this period a correction could be found with some approximation to be applied to the mean error of the Moon's place. This only served for correcting the errors of the lunar tables of long period. The effect of these errors on the terms of short period will be insensible, except for terms like the parallactic inequality, whose arguments have always the same value when they transit at the same hour, and, consequently, in which these errors accumulate. The only term really needing correction is the parallactic inequality. The examination of the observations made during 1864, 1870, and 1877, showed that the clock-errors used for determining the place of the Moon at the transit of the limb at the time of first Quarter were derived from stars which mainly transited six hours earlier than the stars from which was derived the clock-error used for observations of the Moon at the time of third Quarter. The interval was found to be rather less in summer and rather more in winter. Therefore, if during this interval the systematic errors in the Right Ascensions varied, the effect of this variation would be thrown on the apparent value of the parallactic inequality; knowing the amount of the systematic errors in the places of the clock stars, it was easy to calculate their effect on the parallactic inequality, on the annual equation, on the motion in mean longitude, and on the error in mean longitude.

During the years 1862 to 1869, the star places used were derived from the First Seven Year Catalogue, and the values of the systematic errors of this catalogue give the corrections

$$\begin{aligned} \delta L &= +0''.15 \\ \delta(l) &= -0.62 \\ \delta M &= +0.15 \\ \delta P &= +0.06 \end{aligned}$$

During the years 1870–1877, the Second Seven Year Catalogue was used, and the corrections are :—

$$\begin{aligned} \delta L &= +0''.06 \\ \delta(l) &= -0.33 \\ \delta M &= +0.11 \\ \delta P &= +0.04 \end{aligned}$$

These are the quantities which must be added to those derived from the observations in order to correct the observed values for the effect of these systematic errors in the places of the Greenwich clock stars. It will be observed that the most important effect is produced on the apparent motion of the mean longitude of the Moon, an effect amounting to nearly two-thirds of a second during the earlier period. This correction is distinguished as $\delta(l)$, for it is not truly a correction to the motion in mean longitude, but is really a periodical yearly inequality affecting it, so that, though it will be confounded with the apparent motion in mean longitude when this is derived separately for each year, it will vanish from it if the correction to the motion in mean longitude be derived from two or more years combined; but in this case the observations must be corrected for this inequality separately. The effect on the parallactic inequality is insignificant when the observations of a whole year are summed. It is, however, necessary to determine the effect for each quarter of the year, in order to ascertain whether they give rise to different errors in summer and winter. The result is as follows :—

Mean Period for which the correction is derived.	First Seven Year Catalogue.	Second Seven Year Catalogue.
January, February, March,	$\delta P = -0''.09$	$\delta P = -0''.05$
April, May, June,	$= -0''.09$	$= -0''.08$
July, August, September,	$= -0''.35$	$= -0''.18$
October, November, December,	$= +0''.77$	$= +0''.47$

Consequently

Correction to the mean Winter observations	$= +0''.34$	$= +0''.21$
Correction to the mean Summer observations	$= -0''.22$	$= -0''.13$

The very important correction due to this cause is apparent, and it will give rise to differences in the apparent value of the variation in the irradiation amounting to the difference between the above values multiplied by two, or

$$\delta I = -1''.12 \qquad = -0''.68$$

The values thus found for these corrections may be regarded as close approximations to the truth, as making any permissible variations in the assumed basis of the investigation will not alter the value found for the correction by more than a very small amount.

In dealing with these corrections, it is proposed to apply them to the final result for each quarter, and not to the separate observations. This method will effect considerable saving in labour, and give results sensibly as correct.

The third correction to be considered is that denoted by

$$s\delta S$$

the correction to the assumed semi-diameter. This is scarcely a correction to the tables, as every Observatory will have a systematically different value for the apparent semi-diameter, and the correction, like the last two, is better applied to the observed place than to the tabular place of the Moon. The correction for the assumed error in the tabular place of the Moon will really only affect the parallactic inequality, and will be applied to the results, and not to the separate observations themselves. No sensible error can be introduced by this means. The value of δS adopted will be taken from the observed times of transit of the Moon's diameter when both limbs are observed at Full. The corrections are

From 11 observations	between 1862 and 1865	$\delta S =$	$+0''.42$
„ 11 „	1866 „ 1869	$=$	$+0''.33$
„ 11 „	1870 „ 1877	$=$	$+0''.10$

The effect of personality in the transit diameter of the Moon seems to have been over-estimated. The results for the principal Greenwich observers are as follows:—

Dunkin	4 observations	$\delta S =$	$+0''.09$
Ellis	7 „	$=$	$+0''.10$
Criswick	11 „	$=$	$+0''.25$
Lynn	3 „	$=$	$+0''.22$
J. Carpenter	3 „	$=$	$+0''.92$
Thackeray	2 „	$=$	$+0''.75$

The two last are considerably greater that the others, it is true, but the results rest on so very few observations—three and two—that these differences may be said to be accidental. The actual effects on the observed values of the inequalities amount of course to only a small fractional part of these differences.

The principal new term to be introduced is that denoted by

$$\delta K \sin k.$$

It is the inequality due to the direct perturbing action of *Jupiter*, and corresponding to the inequality arising from the action of the Sun known as the Evection. It may be termed, therefore, the Jovian Evection. It is an inequality which has a most important effect on the apparent value of the parallactic inequality, though now for the first time taken into consideration.

This term was discovered in the year 1876 by Prof. Newcomb

as an empirical inequality existing in the mean longitude of the Moon, and arising from an unknown source. Its origin was identified by Mr. Neison, and its value independently calculated by him. Although its existence has long been known, its value seems to have first been calculated by Prof. Hansen, who published his results in his *Darlegung*. He found it to possess a very small coefficient, and, regarding it as insensible, did not include it in his tables. Its true value seems to have been first discovered and made known by the Rev. W. E. Penny, and to this gentleman belongs all the credit attaching to priority. Although the first to be made known, it is not the only term with a sensible coefficient arising from the action of the planets which do not find the place they ought in Hansen's tables. In this respect the lunar tables are decidedly faulty.

The argument of this term can be written in the form

$$k = \alpha + 20^{\circ}\cdot 655[\mathrm{Y} - 1873\cdot 25],$$

so that it may be regarded as a long inequality—with a period of 18 years—in the eccentricity and longitude of the perigee of the lunar orbit. It was in this form that it was discovered by Prof. Newcomb. But it also takes the form of an apparent inequality in the mean longitude of the Moon, and therefore of a yearly inequality in the value of the parallactic inequality. For

$$\alpha = l - \mathrm{A} = l - 40^{\circ}\cdot 675[\mathrm{Y} - 1865\cdot 25],$$

or

$$k = l - 20^{\circ}\cdot 020[\mathrm{Y} - 1875\cdot 00].$$

Now, the argument of the parallactic inequality may be written in the form

$$\theta = l - (\mu + \mathrm{A}) = l - \mu - 281^{\circ}\cdot 2;$$

consequently by substitution

$$k = \theta + \mu - 20^{\circ}\cdot 020[\mathrm{Y} - 1872\cdot 05].$$

This term may now be regarded as an inequality in the value of the parallactic inequality with the value

$$\delta \mathrm{P} = -\delta k \cos \{\mu - 20^{\circ}\cdot 020[\mathrm{Y} - 1872\cdot 05]\}.$$

When the results for a whole year are taken, and the observations are equally distributed over the lunation, the effects of this term will vanish as the positive and negative values will destroy

each other. In practice they will never quite destroy each other, as the observation can only be equally spread over two-thirds of the lunation, so that for some years this term will make the parallactic inequality slightly too large, and for another similar period slightly too small.

But when the observations are divided into half-yearly groups, this will no longer be the case. The value of μ may be taken at 0° at the beginning of the year, and uniformly increased nearly 1° per day, until it becomes 360° on December 31. Now, in 1872 the value of the second term may be considered as zero. Consequently from October to March the argument of the inequality in the value of the parallactic inequality will be between 270° and 90°, or the inequality will be constantly positive, and will make the value of the parallactic inequality too large by $0{\cdot}70\,\delta k$; but from April to September exactly the reverse will happen, and the parallactic inequality be $0{\cdot}70\,\delta k$ too small. When, therefore, these two values are subtracted to obtain the supposed effect of the variation in semidiameter, the variation will be too small by $1{\cdot}40\,\delta k$, a quantity of more than $2''{\cdot}0$. On the other hand, in the year 1863, when the slowly varying factor has increased by 180°, these things will be exactly reversed. There will result, therefore, a periodical variation in the winter and summer values of the parallactic inequality, with a period of eighteen years; and consequently a similar inequality will appear in the value of the irradiation of the limb. The importance of this term, and the fallacy of assuming that the errors of the lunar tables must eliminate themselves in the mean results of a few years, are thus strikingly illustrated.

There only remains to be considered the pair of terms given under the form

$$\delta \mathrm{F} \sin f = \delta \mathrm{F} \sin \{2\theta - \alpha + \mu - \mathrm{N}\},$$
$$\delta \mathrm{F}' \sin f' = \delta \mathrm{F}' \sin \{\alpha - \mu + \mathrm{N}\};$$

where N is supposed to be an argument of long period increasing by only a few degrees per annum, or else differing in its period from that of the annual equation by only a few degrees per annum.

These two terms are intimately connected with each other and with the parallactic inequality. For the arguments of these terms may be written in the form

$$f = (2l - 2l') - (l - \mathrm{A}) + (l' - \mathrm{A}') - \mathrm{N} = (l - l') + (\mathrm{A} - \mathrm{A}') - \mathrm{N} = \theta + (\mathrm{A} - \mathrm{A}' - \mathrm{N}),$$
$$f' = (l - \mathrm{A}) + l' - \mathrm{A}') - \mathrm{N} \qquad = (l - l') - (\mathrm{A} - \mathrm{A}') + \mathrm{N} = \theta - (\mathrm{A} - \mathrm{A}' - \mathrm{N}).$$

Therefore, putting for brevity

$$f'' = \mathrm{A} - \mathrm{A}' - \mathrm{N},$$

these three terms may be written in the form

$$\begin{aligned}\delta P \sin p &= \delta P \sin \{\theta\}\\ \delta F \sin f &= \delta F' \sin \{\theta + f''\}\\ \delta F' \sin f' &= \delta F' \sin \{\theta - f''\}.\end{aligned}$$

Consequently,

$$\begin{aligned}&\delta P \sin p + \delta F \sin f + \delta F' \sin f'\\ &= \{\delta P + (\delta F + \delta F') \cos f''\} \sin \theta + \{(\delta F - \delta F') \sin f''\} \cos \theta.\end{aligned}$$

When, therefore, the coefficient of the argument $\sin \theta$ is determined from the observations, instead of obtaining P, the correction to the parallactic inequality, there will be obtained the quantity

$$\delta P + (\delta F + \delta F') \cos f''.$$

Consequently, if f'' be an argument of long period, there will arise in the value of δP a variation with a period equal to that of f'', but this variation will not affect in any sensible manner the difference between the winter and summer values of the parallactic inequality, or, therefore, the correction for the variation in irradiation. If f'' be an argument of shorter period and differing little from the period of the annual equation, so that $f'' = \mu \pm f'''$—where f''' is an argument of long period—then it will give rise to no sensible variation in the annual values for the correction δP, but will give rise to systematic differences between the winter and summer values, and so produce an apparent inequality of the long period f''' in the value found for the variation in the irradiation.

There exist a number of these terms in the analytical expression for the true longitude of the Moon, all due to the perturbing action of the planets, and many having coefficients which may prove to be of sensible amount. It is obvious, however, that there will be no need to introduce these terms into the equations for the separate observations, when it is intended merely to determine the value of the correction to the parallactic inequality, for it will be quite sufficient to apply to the value found the proper corrections arising from these terms. The application of these corrections must be deferred, moreover, until the values of these coefficients have been more accurately determined.

The principal term in the equation of the centre, or the term with the argument α, will be a term in which any error in the coefficient will be thrown as a systematic variation on the value found for the irradiation. For the argument of this term can be written in the form

$$\alpha = \theta + \mu - (A - A').$$

Strictly, therefore, a correction should be applied to the observation for the effect of any error in this term. But we have already introduced a term with a very similar argument—namely,

$$k = \theta + \mu + (A + A' - 2J),$$

which does not differ much, as

$$A - 2J = -\frac{1}{2}A$$

very nearly. If, therefore, we employ for the coefficient of the argument k the value resulting from observation, it will embody in it very nearly the combined effect of the proper coefficient of the term with the argument k and the correction required by any error in the value of the equation of the centre. If, therefore, this coefficient be derived from observation and substituted in the equation to δP, it will serve, not only to correct the value for the missing term $\delta K \sin k$, but for any error in the value assigned by Hansen to the lunar eccentricity and longitude of perigee. On the other hand, the coefficient δK will not be constant from year to year. The same remark applies likewise to terms having very nearly the same period as the variation and annual equation.

The entire number of nearly 600 observations was now reduced to the form given above and was divided into yearly groups in the same manner as before. Each group was in turn divided into a summer and winter group, with the view of enabling the effect of the variation in irradiation to be determined. The resulting equations were then summed up in the usual manner, and after eliminating δL, the error in mean longitude, the value of δP was determined in terms of the errors of the tables and the indeterminate corrections δV, δM, δK, δl, δI, and the known corrections derived from δA, δT, δS. The results are given below.

In the expressions for δP, the first term is that due solely to the error of the tables; then follows the correction to this quantity required by the error in the term $\delta A \sin a$, and expressed in seconds of arc, so that (δA) is to be regarded as unity; next in order follow the corrections due to the terms $t\delta T$ and $s\delta S$, also expressed in seconds of arc, so that in these terms likewise (δT) and (δS) are to be considered unity. After these follow the corrections depending on δV, δM, δK, δI, δl. It must be remembered that though the coefficients of these indeterminate quantities may seem small, they multiply quantities which may exceed $2''{\cdot}0$.

In the expressions for δI, a similar arrangement is followed.

Summer II. Division.

1876	21 Obs.	$\delta P = -0''.03$	$-0''.10$ (δA)	$-0''.12$ (δT)	$+0''.10$ (δS)	$+0''.04$ δV	$-0''.29$ δM	$-0''.34$ δK	$-0''.73$ δI	$-0''.11$ δl
1875	22	$= -1''.79$	$-0''.06$	$-0''.12$	$+0''.10$	$+0''.27$	$-0''.11$	$-0''.15$	$-0''.66$	$-0''.02$
1874	31	$= -2''.81$	$+0''.10$	$-0''.12$	$+0''.10$	$+0''.07$	$-0''.17$	$+0''.02$	$-0''.78$	$-0''.17$
1873	17	$= -1''.98$	$-0''.12$	$-0''.14$	$+0''.10$	$+0''.09$	$-0''.28$	$+0''.35$	$-0''.76$	$-0''.10$
1872	17	$= -3''.32$	$+0''.06$	$-0''.13$	$+0''.10$	$+0''.23$	$-0''.27$	$+0''.65$	$-0''.77$	$-0''.06$
1871	16	$= -3''.05$	$-0''.03$	$-0''.15$	$+0''.10$	$+0''.10$	$-0''.06$	$+0''.74$	$-0''.82$	$-0''.08$
1870	16	$= -2''.08$	$-0''.02$	$-0''.14$	$+0''.10$	$+0''.40$	$-0''.07$	$+0''.64$	$-0''.70$	$-0''.01$
		Mean $= -2''.15$	$-0''.02$	$-0''.13$	$+0''.10$	$+0''.17$	$-0''.18$	$+0''.27$	$-0''.75$	$-0''.08$

Summer I. Division.

1869	15	$= -0''.52$	$+0''.23$	$-0''.30$	$+0''.33$	$+0''.25$	$-0''.41$	$+0''.27$	$-0''.58$	$-0''.13$
1868	29	$= -1''.56$	$+0''.14$	$-0''.24$	$+0''.33$	$+0''.16$	$-0''.13$	$+0''.22$	$-0''.66$	$-0''.10$
1867	18	$= -2''.72$	$+0''.04$	$-0''.25$	$+0''.33$	$+0''.17$	$-0''.04$	$+0''.05$	$-0''.65$	$+0''.02$
1866	18	$= -2''.08$	$-0''.16$	$-0''.21$	$+0''.33$	$-0''.03$	$-0''.33$	$+0''.02$	$-0''.79$	$-0''.14$
1865	16	$= -2''.72$	$+0''.02$	$-0''.22$	$+0''.42$	$+0''.18$	$-0''.06$	$+0''.02$	$-0''.75$	$-0''.02$
1864	20	$= -2''.25$	$-0''.14$	$-0''.23$	$+0''.42$	$-0''.05$	$-0''.14$	$-0''.51$	$-0''.78$	$-0''.14$
1863	13	$= -0''.99$	$+0''.19$	$-0''.23$	$+0''.42$	$+0''.22$	$-0''.37$	$-0''.34$	$-0''.75$	$-0''.21$
		Mean $= -1''.84$	$+0''.05$	$-0''.24$	$+0''.37$	$+0''.13$	$-0''.21$	$-0''.05$	$-0''.71$	$-0''.10$

Winter II. Division.

1876–1875	14 Obs.	δP = −4″·72	+0″·33 (δA)	+0″·25 (δT)	+0·10 (δS)	+0·00 δV	−0·16 δM	+0·34 δK	−0·32 δI	−0·35 δl
1875–1874	13	= −0·73	+0·23	+0·23	+0·10	−0·07	−0·01	−0·23	−0·29	−0·10
1874–1873	21	= +1·02	−0·04	+0·17	+0·10	−0·00	−0·17	−0·40	−0·29	−0·17
1873–1872	14	= −0·01	+0·02	+0·25	+0·10	+0·10	−0·44	−0·43	−0·21	−0·22
1872–1871	23	= +0·05	+0·08	+0·17	+0·10	+0·03	−0·07	−0·78	−0·15	+0·00
1871–1870	22	= −0·23	−0·14	+0·33	+0·10	+0·03	−0·07	−0·53	−0·24	−0·09
1870–1869	15	= −1·86	+0·08	+0·50	+0·10	−0·10	+0·20	−0·77	−0·32	+0·03
		Mean = −0·93	+0·08	+0·27	+0·10	−0·00	−0·07	−0·40	−0·26	−0·13

Winter I. Division.

1869–1868	20	= −2·35	−0·13	+0·34	+0·33	+0·22	−0·13	−0·58	−0·16	−0·10
1868–1867	18	= −3·02	+0·03	+0·34	+0·33	+0·18	−0·09	−0·24	−0·37	−0·05
1867–1866	20	= −0·90	−0·03	+0·47	+0·33	+0·13	+0·14	−0·24	−0·20	−0·25
1866–1865	18	= −0·95	+0·04	+0·39	+0·33	+0·27	−0·15	−0·10	−0·21	−0·03
1865–1864	21	= −1·93	+0·09	+0·40	+0·42	−0·10	+0·08	+0·30	−0·27	+0·03
1864–1863	18	= −1·72	−0·01	+0·34	+0·42	+0·10	−0·16	+0·58	−0·13	+0·13
1863–1862	21	= −2·25	−0·06	+0·46	+0·42	+0·19	−0·35	+0·49	−0·19	−0·36
		Mean = −1·87	−0·01	+0·39	+0·37	+0·14	−0·07	+0·03	−0·22	−0·09

Irradiation II. Division.

1876–1875	35 Obs.	$\delta I = +11''.92$	$-1''.09$ (δA)	$-0''.96$ (δT)	$+0.09\ \delta V$	$-0.32\ \delta M$	$-1.73\ \delta K$	$+0.70\ \delta l$
1875–1874	35	$= -\ 2.76$	−0.85	−0.94	+0.88	−0.37	+0.20	+0.20
1874–1873	52	$= -\ 7.96$	+0.30	−0.62	+0.14	+0.06	+0.87	+0.02
1873–1872	31	$= -\ 3.61$	−0.27	−0.74	−0.04	−0.08	+1.40	+0.22
1872–1871	40	$= -\ 5.22$	−0.03	−0.50	+0.30	−0.33	+2.29	−0.10
1871–1870	38	$= -\ 4.75$	+0.19	−0.80	+0.13	+0.03	+2.12	+0.02
1870–1869	31	$= -\ 0.56$	−0.27	−1.76	+1.37	−0.72	+3.82	−0.12
		Mean $= -\ 1.85$	−0.29	−0.90	+0.42	−0.25	+1.29	+0.14

Irradiation I. Division.

1869–1868	35	$= +\ 4.35$	+0.86	−1.60	+0.04	−0.61	+2.05	+0.09
1868–1867	47	$= +\ 5.06$	+0.40	−2.20	−0.07	−0.12	+1.67	−0.20
1867–1866	38	$= -\ 4.03$	+0.16	−1.74	+0.09	−0.41	+0.66	+0.66
1866–1865	36	$= -\ 1.88$	−0.33	−1.06	−0.51	−0.39	+0.21	−0.20
1865–1864	37	$= -\ 1.68$	−0.15	−1.36	+0.60	−0.32	−0.59	−0.12
1864–1863	38	$= -\ 0.74$	−0.20	−0.84	−0.24	−0.19	−1.64	−0.42
1863–1862	34	$= +\ 2.22$	+0.44	−1.26	+0.04	−0.04	−2.01	+0.26
		Mean $= +\ 0.47$	+0.17	−1.44	−0.01	−0.30	+0.05	−0.02

In the equations for determining δI, if each year be given equal weight as in the preceding, it is obvious that the years for which the coefficient of δI is least when the winter values of δP are subtracted from the summer values will have greater weight really assigned to them than those for which the coefficient of δI is greatest. But this is exactly the reverse to what is desirable. For this reason, instead of using the mean of the result for each year, it will be more satisfactory to group the whole seven years together and determine δI from the resulting equation. In this manner each year will have its proper weight assigned to it. The results of this solution are

II. Division.

1876–1869	262 Obs.	$\delta I = -2''.53$	$-0''.20\ (\delta A)$	$-0''.86\ (\delta T)$	$+0.35\ \delta V$	$-0.22\ \delta M$	$+1.39\ \delta K$	$+0.10\ \delta l$

I. Division.

1869–1862	265 Obs.	$\delta I = +0''.07$	$+0''.11\ (\delta A)$	$-1''.33\ (\delta T)$	$+0.04\ \delta V$	$-0.30\ \delta M$	$-0.23\ \delta K$	$-0.03\ \delta l$

Uniting the summer and winter portions of each division together, the value for the correction to the parallactic inequality becomes

II. Division.

1876–1869	262 Obs.	$\delta P = -1''.54$	$+0''.03\ (\delta A)$	$+0''.07\ (\delta T)$	$+0''.10\ (\delta S)$	$+0.08\ \delta V$	$-0.12\ \delta M$	$-0.06\ \delta K$	$-0.50\ \delta I$	$-0.10\ \delta l$

I. Division.

1869–1862		$\delta P = -1''.85$	$+0''.02\ (\delta A)$	$+0''.07\ (\delta T)$	$+0''.37\ (\delta S)$	$+0.14\ \delta V$	$-0.14\ \delta M$	$-0.15\ \delta K$	$-0.46\ \delta I$	$-0.10\ \delta l$

An examination of the coefficient of the different corrections to the coefficient of the various inequalities shows the important part they take in creating apparent discordances in the value found for the coefficient of the parallactic inequality, and the necessity which exists for their being taken into account in any investigations. It so happens that in the two seven-year groups these corrections to a great extent destroy each other. Yet this is merely accidental, and the mere shifting of a year from one group to another would make important alterations. Had the period of fifteen years been divided into three five-year groups the resulting values would be utterly discordant unless these terms were introduced which arise from the errors in Hansen's tables. As previously pointed out, the most important term is that due to the disturbing action of the planet *Jupiter*, and known as the Jovian Evection. In the expression for the variation in the irradiation the coefficient multiplying this correction varies between $-2\cdot0$ and $+3\cdot8$, and as the correction amounts to $1''\cdot5$, this corresponds to a variation in the apparent value of the irradiation of from $-3''\cdot0$ to $+5''\cdot7$, or a range of $8''\cdot7$.

It remains to consider the value of the corrections denoted by δV, δK, δK, δl, which must be employed for determining the true value of the coefficient of the parallactic inequality. The values employed will be derived from a discussion of the entire mass of meridian observations made at Greenwich between 1862 and 1877. Only provisional corrections have as yet been deduced but it is most unlikely that any changes which may be required when the theory of the terms of long period is placed on a satisfactory footing will affect in any important degree the value of the parallactic inequality in so far as it depends on these terms. These provisionally adopted values are

$$\delta V = +0''\cdot25$$
$$\delta M = -1\cdot20$$
$$\delta K = +1\cdot50$$
$$\delta l = +1\cdot00$$

Employing these results, we have

$$\text{II. Division} \quad \delta I = -1''\cdot06 \pm 0''\cdot25,$$
$$\text{I. Division} \quad \delta I = -1\cdot18 \pm 0\cdot20;$$

whence, uniting the two groups,

$$\delta I = -1''\cdot12 \pm 0''\cdot14.$$

Next, considering the value of the apparent coefficient of the parallactic inequality, we have

$$\text{II. Division} \quad \delta(P) = -1''\cdot37 \pm 0''\cdot06,$$
$$\text{I. Division} \quad \delta(P) = -1\cdot33 \pm 0\cdot08;$$

the mean of the two divisions is

$$\delta(P) = -1''.35 \pm 0''.05.$$

Lastly, taking the value of the true coefficient of the parallactic inequality, we have

II. Division $\delta P = -1''.37 - 0''.50\,\delta I = -0''.84 \pm 0''.15,$

I. Division $\delta P = -1.33 - 0.46\,\delta I = -0.80 \pm 0.12.$

The mean of these two very accordant values gives us the final value for the real coefficient of the parallactic inequality,

$$\delta P = -0''.82 \pm 0''.09.$$

From these corrections we deduce the results :—

True value of the apparent coefficient

$$(P) = -125''.11 \pm 0''.05.$$

True value of the real coefficient

$$P' = -125''.64 \pm 0''.09.$$

Comparing these with the results obtained by the previous approximate solution, which were

$$(P) = -125''.06 \pm 0''.10,$$
$$P = -125.66 \pm 0.34,$$

it will be seen that they do not differ materially, but the weights to which each is entitled are incomparably different.

The above value for the real coefficient of the parallactic inequality we present as the final result of the investigation, and we regard it as the most satisfactory result which has as yet been obtained, it being the only value which has been derived strictly from the observations without the introduction of any empirical correction.

It must be admitted, however, that this result has still to be corrected for the influence of any inequality of long period which may exist in the apparent value of the parallactic inequality and arise from the perturbations produced by the planets. It has already been shown that if N be an argument of long period increasing by only a few degrees per annum, there will be two classes of terms which will seriously affect the apparent value of the coefficient of the parallactic inequality—those with the argument

$$\theta + N,$$

which will produce directly a long inequality in the value of P, but will not sensibly affect the apparent value of δI, the coeffi-

cient of the variation in irradiation, and those with the argument

$$\theta + \mu + N,$$

which will only indirectly affect the value of P by producing an inequality of long period in the apparent value of δI, and thus to a smaller degree in P.

As far as the first class of terms are concerned : if the observations be spread over many years they could be determined from observation without much difficulty, as they will affect the value of (P) and of P in the same manner. If, therefore, they were otherwise than small, it might be anticipated that they would be revealed by an examination of the values derived from the Greenwich observations made between 1750 and 1851. With regard to the second class: the only material which exists for their investigation is the values found above for δI from the observations between 1862 and 1877. These are insufficient; they would suffice to detect any inequality of eight or nine years' period, but such a term is unimportant, as its effect would have been eliminated by the method adopted for determining P. A term of much longer period may exist, but the materials do not exist for recognising it.

The proper method for determining the effect of such terms is unquestionably by a thorough investigation of the theory of the perturbations in the motion of the Moon due to the disturbing action of the planets—a work now far advanced in the hands of one of us. Beyond the analytical labour there is no difficulty in the determination of the value of every inequality due to the action of the planets which *could* possibly exceed $0''{\cdot}01$. This effected, there will vanish the last theoretical difficulty in the way of determining the solar parallax by means of the parallactic inequality. Until it be done, some uncertainty must still remain.

When the values assigned to the correction $\delta V, \delta M, \delta K, \delta l$ come to be substituted in the expressions for δI during the different years, it will be found that there still remain considerable outstanding errors. These are partly due to the fact that constant values have been assigned to these corrections, whereas the observations show that they are subject to some variation due to the fact that Hansen's tables involve one or two more corrections not included in the preceding expression. Strictly speaking, in each year should have been used the values found for $\delta V, \delta M, \delta K, \delta l$ from the observations of that year, as in that manner these neglected corrections would have been practically eliminated, The method actually adopted for determining the value of P gives results which are sensibly as accurate as if the stricter method had been employed, for the corrections are multiplied by such small coefficients that slight variations may be neglected. When the expressions for δI are considered the use of the stricter method markedly reduces the values of the outstanding errors in the years 1876, 1869, 1868. But it must be remembered

that the apparent errors of these years are much exaggerated by their being derived from equations in which δI is multiplied by a small coefficient, so that in dividing by this coefficient the apparent errors are increased to double or triple or even quadruple their true amount. In the value for the irradiation which has been derived from the observations and actually employed, all the equations are grouped together and appear with their proper weight according to the largeness of their coefficients of δI. The result, therefore, is entitled to far more weight than may appear from a consideration of the outstanding discordances, so many of which have been thus exaggerated.

It remains to compare together the different results, with the view of seeing how far they harmonise with each other. However, it will first be necessary to consider whether the values found by Airy, Stone, and Newcomb require any further corrections to those already deduced.

The term δK sin k requires first consideration. As this term produces an eighteen-year inequality in the value of P, it may be expected to produce a periodical variation in the values found by Sir G. Airy from his nine-year groups. This anticipation proved true, and it was found that the existence of this term in the Moon's true longitude will necessitate a correction approximately amounting to $0''{\cdot}20$ being applied with alternate signs to every pair of values in Sir G. Airy's list, commencing with the groups for which the mean years are 1847 and 1843. If this correction be applied, it will be found to markedly reduce the outstanding residuals in the comparison between observation and the theory of a forty-five-year inequality. This is shown by the following figures, where in the first line are placed the uncorrected results for the last ten groups, and in the second line the corrected results, found by applying $\pm 0''{\cdot}20$ to every alternate pair :—

$$-0''{\cdot}24+0''{\cdot}10-0''{\cdot}28-0''{\cdot}25-0''{\cdot}32+0''{\cdot}35+0''{\cdot}06-0''{\cdot}03-0''{\cdot}21+0''{\cdot}31$$
$$-0{\cdot}04-0{\cdot}10+0{\cdot}08-0{\cdot}05-0{\cdot}12+0{\cdot}15-0{\cdot}14+0{\cdot}17-0{\cdot}01+0{\cdot}11.$$

The sum of the residuals in the first line is $2''{\cdot}15$, whilst those in the second line only amount to $0''{\cdot}97$, showing a quadruple weight. The existence of this term δK sin k does not seem to otherwise affect Sir G. Airy's values.

From the omission of this term δK sin k from Hansen's tables, and from the existence in the tables of an error of the form δA sin a, it is obvious that the results obtained by Mr. Stone and by Prof. Newcomb will require correction for these imperfections in the tables. For Mr. Stone's value the corrections can be only approximately formed by assuming that the distribution over the lunation of the observations employed by him did not materially differ from the results for the Greenwich observations during the last fifteen years. From this basis it is

easily shown by the variation in the value of K, that Mr. Stone's result will require the correction

$$+\frac{1}{5}\delta K + 0\cdot05\,\delta A.$$

The correction required by Mr. Newcomb's result can be determined with more certainty by a comparison of the Washington Observations with the results obtained above from the discussion of the Greenwich Observations. This comparison indicates that Prof. Newcomb's value requires the correction

$$-\frac{1}{9}\delta K - 0\cdot15\,\delta A.$$

The result obtained by Mr. Stone will probably also require correction for the systematic errors in the Greenwich Twelve Year Catalogue. This correction will amount to

$$+0''\cdot18.$$

Employing these results, the different values which have been obtained for the apparent coefficient of the parallactic inequality may be written in the form

Airy $\quad (P) = -124''\cdot53 + \frac{1}{3}B,$

Stone $\quad (P) = -124''\cdot55 - \frac{1}{7}B + \frac{1}{9}\delta V - \frac{1}{7}\delta M + \frac{1}{5}\delta K + 0\cdot05\,\delta A + 0''\cdot18,$

Newcomb $\quad (P) = -124''\cdot36 + \frac{2}{5}B + \frac{1}{9}\delta V - \frac{1}{7}\delta M - \frac{1}{9}\delta K - 0\cdot15\,\delta A,$

C. & N. $\quad (P) = -125''\cdot03 + \frac{4}{5}B.$

Substituting for the different corrections, except B, the values which have been assigned to them, we have

Airy $\quad (P) = -124''\cdot53 + \frac{1}{3}B,$

Stone $\quad (P) = -123''\cdot84 - \frac{1}{7}B,$

Newcomb $\quad (P) = -124''\cdot42 + \frac{2}{5}B,$

C. & N. $\quad (P) = -125''\cdot11 + \frac{4}{5}B.$

It is obvious that if B be neglected, or supposed zero, which corresponds to assuming that the hypothetical forty-five-year inequality is non-existent, the resulting values for the apparent coefficient of the parallactic inequality are far from accordant.

If, however, for B be substituted the value derived from Sir G. Airy's investigations,

$$B = +1''.20 \pm 0''.20,$$

the different values become

Airy	(P) =	$-124''.53+0''.40 = -124''.13$,
Stone	(P) =	$-123.84-0.17 = -124.01$,
Newcomb	(P) =	$-124.42+0.48 = -123.96$,
C. & N.	(P) =	$-125.11+1.00 = -124.11$.

These results are extremely accordant, probably accidentally so, the mean of the four being

$$(P) = -124''.05.$$

It is obvious that the altogether unexpected coincidence of the values tends to very materially strengthen the hypothesis of the real existence of this inequality,

$$1''.20 \sin \{\theta - 8.0(Y - 1825.5)\},$$

which so markedly renders them accordant. But for one circumstance, this marvellous accord would render the existence of this inequality unquestionable. This circumstance is the fact that it is the apparent values which are thus rendered so accordant, and there is no reason why these values should be accordant. For they depend on the variation of the irradiation at the limb, and the amount of this variation may differ in different instruments and with different observers, so that, although the real values of the coefficient of the parallactic inequality must agree, there is no such reason why the apparent values should thus agree. There would be nothing remarkable in the values for different observations differing by a considerable fraction of a second. This consideration must prevent too great reliance being placed on this accord.

The final conclusion of our investigation must be stated as follows. That the real value of the parallactic inequality in the motion of the Moon is either

$$-125''.64 \pm 0''.09$$

or

$$-124''.64 \pm 0''.25.$$

Which value is correct will depend on whether we admit or do not admit the existence of this forty-five-year inequality. Now, does theory admit of the existence of such a term? Sir G. Airy replies in the negative. But this is, we conceive, an oversight.

In the theory of the perturbation of the Moon produced by the action of the planets, there unquestionably exist terms with arguments of this form and, moreover, with coefficients which *may* amount to a second of arc. There is, therefore, nothing inconsistent with theory in the existence of such a term. The question, Does it exist? is one which cannot be answered until the theory of these terms has been further advanced, and this does not come within the scope of our present paper.

It must be admitted that this uncertainty is not satisfactory; but, on the other hand, it is not discouraging. In the present paper it has been shown that it is possible to derive a fairly satisfactory result for the value of the parallactic inequality from the existing observations. It only remains to effect an auxiliary investigation to enable the true value of the parallactic inequality to be deduced from the results obtained in the present paper. This is the investigation of those neglected inequalities in the motion of the Moon due to the action of the planets. This effected—and it is merely a work of moderate labour, and does not present any particular difficulty—the results just obtained will enable us to definitely determine the value of the solar parallax from the parallactic inequality.

In the meantime we think it well to submit the present investigation to the Society, as it is really a work complete in itself, and one which has led to novel and important results.

It remains to determine the value of the solar parallax from the value found for the coefficient of the parallactic inequality. For this purpose Newcomb's data will be taken. Let

π = constant of solar parallax,

Π = constant of lunar parallax = $3422''.7$,

μ = mass of the Moon = $\frac{1}{81.5}$.

m = the ratio of the mean motion of the Sun and Moon,

f = value of analytical coefficient of parallactic inequality.

Then

$$\sin \pi = -\sin \Pi \left(1 - \frac{1}{6} m^2\right) \frac{1+\mu}{1-\mu} f \,.\, \mathrm{P}.$$

The values of the coefficient of P are as follows:—

Hansen = $1 \div 14.189$,
Delaunay = $1 \div 14.198$,
Neison = $1 \div 14.201$.

The value adopted will be

$1 \div 14.200$.

Taking merely, as it stands, the value derived from the Greenwich observations between 1862 and 1877, or

$$P = -125''.64 \pm 0''.09,$$

it gives the value for the solar parallax

$$\pi = 8''.848 \pm 0''.007.$$

If, however, the assumed existence of the forty-five-year inequality be admitted, then the value of the parallactic inequality is

$$P = 124''.64 \pm 0''.25,$$

or the solar parallax is

$$\pi = 8''.778 \pm 0''.018.$$

Which of these is the correct value must be deferred for future investigation.

In the preceding portions of this communication, attention has been confined to the determination of the value of the parallactic inequality, by the discussion of meridian observations of the transit of the lunar limb. It has there been shown that, apart from some remediable imperfections in the theory of the action of the planets upon the motion of the Moon, the whole difficulty of the problem lies in determining the effect of the variations in the apparent semi-diameter of the Moon arising from the variations in the irradiation due to the varying contrast between the brightness of the limb and the darkness of the sky. If this uncertainty were only eliminated, there would remain no difficulty in the way of determining the value of the parallactic inequality to within $0''.05$ of the truth. This would correspond to determining the value of the solar parallax to within $0''.003$.

As long as the observations to be discussed are observations of the limb of the Moon, this difficulty about the determination of the variation in the semi-diameter must be faced. It may be more or less successfully overcome, and we may reduce the uncertainty to within far smaller limits than was heretofore thought possible, but even under the best conditions it will always be a source of some uncertainty. It has long appeared to us, that the right method of facing this problem of determining the true value of the parallactic inequality was to avoid this difficulty altogether, by entirely abandoning the use of the limb. Let the object observed be a small conspicuous craterlet, near the centre of the nearer hemisphere of the Moon, and the entire difficulty about the semi-diameter and its variation utterly vanishes.

Suppose a series of transits of such a craterlet be taken with

a high-class Meridian instrument like the Transit Circles at Greenwich or Washington. These observations can be reduced to the centre of the Moon, with as much ease and far greater certainty than any observation of the limb. It is only necessary to apply the correction depending on the libration of the Moon. This involves no difficulty, for the apparent libration of the Moon can be easily calculated to within $0''{\cdot}02$. For this libration is simply the difference between the true and mean places of the Moon, and an error of $1''$ in the position of the Moon in the heavens, corresponds to an error of only $0''{\cdot}005$ in the apparent libration. The only difficulty which could be supposed to exist lies in the uncertainty as to the amount of any real libration of the Moon. But this cannot exceed $2''{\cdot}0$ for any terms with periods shorter than a year, and is probably far smaller. Even were it much larger, as the period of its variation must differ materially from that of the parallactic inequality, it could not give rise to any sensible error in determining the value of this term. The theory of the motion of the real libration of the Moon is so well known that its effects could be eliminated from the values deduced for all the terms of short period in the lunar theory. The effect of any real libration with a period greater than a year could not affect the determination of the errors in the inequalities of short period. It would, moreover, be easy to make the same series of observations which served to determine the value of the coefficient of the parallactic inequality serve to determine the value of this real libration. The libration of the Moon does not present, therefore, any difficulty in the employment of this method of observation, a method which offers the great advantage of rendering us independent of all the difficulties involved in the uncertainty as to the real semi-diameter of the Moon, or of its variations due to differences in irradiation, or to the existence of irregularities at the limb.

The accuracy of the observations would also be materially improved, for it is far easier to observe a sharply-marked craterlet than to observe the irradiated limb. A craterlet could be selected which would present the appearance of a sharply-defined white ring, about $4''$ in diameter, surrounding a grey interior about $2''$ in diameter. Such an object can be observed with fully twice the accuracy of the limb. The probable error of a single observation of the lunar limb with the fine Transit Circles at Greenwich and Washington has been found to be $2''{\cdot}0$; whereas, even with a small Transit instrument, a four-feet Transit, the probable error of a single observation of a craterlet on the surface is only $\pm 1''{\cdot}00$. In a high-class instrument it would be less. The systematic errors involved in the observation of the transit of such an object would be far less important than those in the case of the Moon's limb, for the observation would be made under practically the same conditions day after day throughout the entire semi-lunation, and so disappear from the result; whereas those of the observation of

the lunar limb change their character at Full Moon, when the limb observed is changed. It would be only at the time of first and third Quarters that any shadows would be present; during the rest of the period, the craterlet would preserve unchanged its appearance as a white circlet with a grey interior on a yellowish surface.

Even apart from the greater accuracy with which the observation could be made, the great advantage of being independent of any uncertainty about the lunar semi-diameter and its variation would render this very method far superior to that founded on observations of the limb.

We believe that five hundred such observations of the Moon spread over eight or nine years would enable us to deduce the true value of the parallactic inequality to within far less than $0''\cdot10$.

To properly carry out this method, it is essential that a permanently mounted Transit Circle of good design should be used. But until this be done, much good might result from the employment of a less powerful instrument. In this manner not only might the advantages of the method be demonstrated but much valuable time be saved. For this reason, therefore, this work has been undertaken at the Arkley Observatory, belonging to Mr. Campbell.

The craterlet selected for observation is that known as Murchison A, a small craterlet about 5° of selenographical arc from the mean centre of the visible hemisphere, and very approximately in $+ 1°0'\cdot1$ selenographical longitude and $+ 4°4'\cdot0$ selenographical latitude. Each day's work, when possible, comprises a transit of Murchison A and the observation of ten or twelve clock stars including four Moon culminating stars. The instrument employed is a Transit instrument of good quality, of $2\frac{3}{4}$ feet focal length and $2\cdot7$ inches' aperture, with a massive iron stand resting on a substantial stone pier. Each observation is reduced by an expansion of the method detailed in chapter xxix. of Neison's work on the Moon. Every observation is then carefully compared with the tabular place of the Moon, and the apparent error of the tables is equated to the assumed error in the parallactic inequality and other terms which may be supposed to need correction. These equations will, when a sufficient number is obtained, serve for determining the different errors in the tabular values assigned by Hansen to the different inequalities in the motion of the Moon.

Owing to the remarkably bad weather of the past year the number of observations obtained have been fewer than was anticipated. The work was commenced in April 1879. During the last year only 21 observations were obtained. All these have been reduced. During the first four months of this year no less than 27 additional observations were obtained. Most of these have been reduced.

It is intended that the results obtained from the discussion

of the observations now in progress at the Arkley Observatory should form a sequel to the present communication, and thus afford a complete solution of the great problem of determining the solar parallax from the parallactic inequality in the motion of the Moon.

Arkley Observatory : Barnet, Herts,
1880, *May* 13.

Investigation of the Secular Acceleration of the Moon's Mean Motion, caused by the Secular Change in the Eccentricity of the Earth's Orbit; taking into Account Terms of the Order of m^4, *but neglecting the Eccentricity and Inclination of the Moon's Orbit.* By Professor J. C. Adams, M.A., F.R.S.

As the question of the Moon's secular acceleration has lately been again brought before the Society, I have thought that it might not be useless or without interest to communicate an investigation of the two leading terms of that acceleration which I gave many years ago in my lectures on the lunar theory.

1. Let r, θ be the polar coordinates of the Moon at time t, $u = \frac{1}{r}$, $\mathrm{H} = r^2 \frac{d\theta}{dt}$, μ the sum of the masses of the Earth and Moon; also let m' be the mass of the Sun, r', θ' its polar coordinates, a' the Sun's mean distance, n' its mean motion, and e' the eccentricity of its orbit, $\lambda' = n't + \varepsilon'$ its mean longitude, and $\phi' = n't + \varepsilon' - \omega'$ its mean anomaly.

Then the equations to be satisfied are

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu}{\mathrm{H}^2} - \frac{1}{2}\frac{m'}{\mathrm{H}^2 u^3 r'^3} - \frac{3}{2}\frac{m'}{\mathrm{H}^2 u^3 r'^3}\cos 2(\theta - \theta') + \frac{3}{2}\frac{m'}{\mathrm{H}^2 u^4 r'^3}\frac{du}{d\theta}\sin 2(\theta - \theta'),$$

and

$$\frac{d(\mathrm{H}^2)}{d\theta^2} = -\frac{3m'}{u^4 r'^3}\sin 2(\theta - \theta').$$

Also, by the formulæ of elliptic motion

$$\frac{m'}{r'^3} = \frac{m'}{a'^3}\left(\frac{a'}{r'}\right)^3 = n'^2\left\{1 + \frac{3}{2}e'^2 + 3e'\cos\phi' + \frac{9}{2}e'^2\cos 2\phi'\right\},$$

$$\frac{m'}{r'^3}\cos 2(\theta - \theta') = \frac{m'}{a'^3}\left(\frac{a'}{r'}\right)^3 \cos 2(\theta - \theta')$$

$$= n'^2\left\{\left(1 - \frac{5}{2}e'^2\right)\cos 2(\theta - \lambda') + \frac{7}{2}e'\cos(2\theta - 2\lambda' - \phi') - \frac{1}{2}e'\cos(2\theta - 2\lambda' + \phi') + \frac{17}{2}e'^2\cos(2\theta - 2\lambda' - 2\phi')\right\},$$

and

$$\frac{m}{r'^3}\sin 2(\theta-\theta') = \frac{m'}{a'^3}\left(\frac{a'}{r'}\right)^3 \sin 2(\theta-\theta')$$

$$= n'^2\left\{\left(1-\frac{5}{2}e'^2\right)\sin 2(\theta-\lambda') + \frac{7}{2}e'\sin(2\theta-2\lambda'-\phi')\right.$$

$$-\frac{1}{2}e'\sin(2\theta-2\lambda'+\phi')$$

$$\left.+\frac{17}{2}e'^2\sin(2\theta-2\lambda'-2\phi')\right\}.$$

The angles involved in these expressions are formed by combining the angle $2\theta-2\lambda'$ with multiples of ϕ'.

For our present purpose we may omit the terms which involve $2\phi'$. Also, for the sake of brevity we may write

$$\begin{aligned}
&n't \text{ instead of } n't+\epsilon'-\omega' \text{ or } \phi',\\
&2\theta-2n't \text{ instead of } 2\theta-2(n't+\epsilon') \text{ or } 2\theta-2\lambda',\\
&2\theta-3n't \text{ instead of } 2\theta-2\lambda'-\phi',\\
&2\theta-\ \ n't \text{ instead of } 2\theta-2\lambda+\phi',
\end{aligned}$$

since no ambiguity can arise from this abbreviation.

Hence our equations become

$$\frac{d^2u}{d\theta^2}+u = \frac{\mu}{H^2}-\frac{1}{2}\frac{n'^2}{H^2u^3}\left\{1+\frac{3}{2}e'^2+3e'\cos n't\right\}$$

$$-\frac{3}{2}\frac{n'^2}{H^2u^3}\left\{\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)+\frac{7}{2}e'\cos(2\theta-3n't)\right.$$

$$\left.-\frac{1}{2}e'\cos(2\theta-n't)\right\}$$

$$+\frac{3}{2}\frac{n'^2}{H^2u^4}\frac{du}{d\theta}\left\{\left(1-\frac{5}{2}e'^2\right)\sin(2\theta-2n't)+\frac{7}{2}e'\sin(2\theta-3n't)\right.$$

$$\left.-\frac{1}{2}e'\sin(2\theta-n't)\right\},$$

and

$$\frac{d(H^2)}{d\theta} = -\frac{3n'^2}{u^4}\left\{\left(1-\frac{5}{2}e'^2\right)\sin(2\theta-2n't)+\frac{7}{2}e'\sin(2\theta-3n't)\right.$$

$$\left.-\frac{1}{2}e'\sin(2\theta-n't)\right\}.$$

2. After these preliminaries, it will be convenient to begin by finding the relations between the actual mean motion n of the Moon and the constant parts of u and H^2 when these quantities are developed in the form we have adopted, carrying the approximation as far as terms involving $m^4e'^2$, on the supposition that e' and therefore also that n is constant.

For this purpose it is sufficient to take

$$nt+\epsilon = \theta + 3me' \sin n't - \frac{11}{8}m^2\left(1-\frac{5}{2}e'^2\right)\sin(2\theta-2n't)$$
$$-\frac{77}{16}m^2e'\sin(2\theta-3n't)+\frac{11}{16}m^2e'\sin(2\theta-n't),$$
$$u = \frac{1}{a}\left\{1-\frac{3}{2}m^2e'\cos n't+m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)\right.$$
$$\left.+\frac{7}{2}m^2e'\cos(2\theta-3n't)-\frac{1}{2}m^2e'\cos(2\theta-n't)\right\},$$

which are readily derived from the equations of motion.

Differentiate the first of these equations and put

$$\frac{n'}{n} = m,$$

$$\therefore \frac{ndt}{d\theta}\left\{1-3m^2e'\cos n't-\frac{11}{4}m^3\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)-\frac{231}{16}m^3e'\cos(2\theta-3n't)\right.$$
$$\left.+\frac{11}{16}m^3e'\cos(2\theta-n't)\right\}$$
$$=1-\frac{11}{4}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)-\frac{77}{8}m^2e'\cos(2\theta-3n't)+\frac{11}{8}m^2e'\cos(2\theta-n't),$$

or

$$\frac{ndt}{d\theta} = 1+\frac{9}{2}m^4e'^2+3m^2e'\cos n't-\frac{11}{4}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)$$
$$-\frac{77}{8}m^2e'\cos(2\theta-3n't)+\frac{11}{8}m^2e'\cos(2\theta-n't),$$

since the other terms only give rise to terms of higher orders than we have here taken into account.

Hence

$$H^2 = \left(\frac{d\theta}{u^2dt}\right)^2 = u^{-4}\left(\frac{dt}{d\theta}\right)^{-2}$$
$$= n^2a^4\left\{1+5m^4(1-5e'^2)+\frac{45}{4}m^4e'^2+\frac{245}{4}m^4e'^2+\frac{5}{4}m^4e'^2+6m^2e'\cos n't\right.$$
$$\left.-4m^3\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)-14m^2e'\cos(2\theta-3n't)+2m^2e'\cos(2\theta-n't)\right\}$$
$$\times\left\{1-9m^4e'^2+\frac{27}{2}m^4e'^2+\frac{363}{32}m^4(1-5e'^2)+\frac{17787}{128}m^4e'^2+\frac{363}{128}m^4e'^2\right.$$
$$-6m^2e'\cos n't+\frac{11}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)+\frac{77}{4}m^2e'\cos(2\theta-3n't)$$
$$\left.-\frac{11}{4}m^2e'\cos(2\theta-n't)\right\};$$

or, by actual multiplication,

$$\begin{aligned}H^2 = n^2a^4\Big\{&1+\frac{523}{32}m^4(1-5e'^2)+\frac{14083}{64}m^4e'^2-18m^4e'^2-11m^4(1-5e'^2)\\&-\frac{539}{4}m^4e'^2-\frac{11}{4}m^4e'^2+\frac{3}{2}m^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2\theta-2n't)\\&+\frac{21}{4}m^2e'\cos(2\theta-3n't)-\frac{3}{4}m^2e'\cos(2\theta-n't)\Big\}\\=n^2a^4\Big\{&1+\frac{171}{32}m^4(1-5e'^2)+\frac{4131}{64}m^4e'^2+\frac{3}{2}m^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2\theta-2n't)\\&+\frac{21}{4}m^2e'\cos(2\theta-3n't)-\frac{3}{4}m^2e'\cos(2\theta-n't)\Big\}.\end{aligned}$$

Hence the constant part of H^2 is

$$n^2a^4\Big\{1+\frac{171}{32}m^4+\frac{2421}{64}m^4e'^2\Big\},$$

n being the actual mean motion.

Hence

$$\begin{aligned}\frac{\mu}{H^2}=\frac{\mu}{n^2a^4}\Big\{&1-\frac{171}{32}m^4(1-5e'^2)-\frac{4131}{64}m^4e'^2+\frac{9}{8}m^4(1-5e'^2)+\frac{441}{32}m^4e'^2+\frac{9}{32}m^4e'^2\\&-\frac{3}{2}m^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2\theta-2n't)-\frac{21}{4}m^2e'\cos(2\theta-3n't)\\&+\frac{3}{4}m^2e'\cos(2\theta-n't)\Big\}\\=\frac{\mu}{n^2a^4}\Big\{&1-\frac{135}{32}m^4(1-5e'^2)-\frac{3231}{64}m^4e'^2-\frac{3}{2}m^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2\theta-2n't)\\&-\frac{21}{4}m^2e'\cos(2\theta-3n't)+\frac{3}{4}m^2e'\cos(2\theta-n't)\Big\},\end{aligned}$$

and therefore the constant part of $\frac{\mu}{H^2}$ is

$$\frac{\mu}{n^2a^4}\Big\{1-\frac{135}{32}m^4-\frac{1881}{64}m^4e'^2\Big\}.$$

3. Also

$$\begin{aligned}\frac{du}{d\theta}=\frac{1}{a}\Big\{&\frac{3}{2}m^3e'\sin n't-2m^2\Big(1-\frac{5}{2}e'^2\Big)\sin(2\theta-2n't)\\&-7m^2e'\sin(2\theta-3n't)+m^2e'\sin(2\theta-n't)\Big\},\end{aligned}$$

and

$$\begin{aligned}\frac{d^2u}{d\theta^2}=\frac{1}{a}\Big\{&-4m^2\Big(1-\frac{5}{2}e'^2\Big)\cos(2\theta-2n't)-14m^2e'\cos(2\theta-3n't)\\&+2m^2e'\cos(2\theta-n't)\Big\};\end{aligned}$$

also

$$\frac{n'^2}{\mathrm{H}^2u^3} = \frac{m^2}{a}\left\{1+\frac{9}{2}m^2e'\cos n't-\frac{9}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)\right.$$
$$\left.-\frac{63}{4}m^2e'\cos(2\theta-3n't)+\frac{9}{4}m^2e'\cos(2\theta-n't)\right\},$$

and

$$\frac{n'^2}{\mathrm{H}^2u^4}\frac{du}{d\theta} = \frac{m^2}{a}\left\{-2m^2\left(1-\frac{5}{2}e'^2\right)\sin(2\theta-2n't)-7m^2e'\sin(2\theta-3n't)\right.$$
$$\left.+m^2e'\sin(2\theta-n't)\right\}.$$

Hence, substituting in the first differential equation and transposing, we find the quantity which is to be equated to $\frac{\mu}{\mathrm{H}^2}$ to be

$$\frac{1}{a}\left\{1+\left(\frac{1}{2}+\frac{3}{4}e'^2\right)m^2+\frac{27}{8}m^4e'^2-\frac{27}{8}m^4(1-5e'^2)-\frac{1323}{32}m^4e'^2-\frac{27}{32}m^4e'^2\right.$$
$$+\frac{3}{2}m^4(1-5e'^2)+\frac{147}{8}m^4e'^2+\frac{3}{8}m^4e'^2$$
$$\left.-\frac{3}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)-\frac{21}{4}m^2e'\cos(2\theta-3n't)+\frac{3}{4}m^2e'\cos(2\theta-n't)\right\}$$
$$=\frac{1}{a}\left\{1+\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)-\frac{15}{8}m^4(1-5e'^2)-\frac{321}{16}m^4e'^2\right.$$
$$-\frac{3}{2}m^2\left(1-\frac{5}{2}e'^2\right)\cos(2\theta-2n't)-\frac{21}{4}m^2e'\cos(2\theta-3n't)$$
$$\left.+\frac{3}{4}m^2e'\cos(2\theta-n't)\right\}.$$

Comparing this with the former expression and observing that $\frac{\mu}{n^2a^3}$ is nearly $=1$, we see that the periodic terms agree, and by equating the non-periodic parts, we have

$$\frac{\mu}{n^2a^3}\left\{1-\frac{135}{32}m^4(1-5e'^2)-\frac{3231}{64}m^4e'^2\right\}$$
$$=1+\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)-\frac{15}{8}m^4(1-5e'^2)-\frac{321}{16}m^4e'^2,$$

or

$$\frac{\mu}{n^2a^3} = 1+\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)+\frac{75}{32}m^4(1-5e'^2)+\frac{1947}{64}m^4e'^2$$
$$=1+\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)+\frac{75}{32}m^4+\frac{1197}{64}m^4e'^2,$$

which gives the relation between n and a.

4. In the above, e' is considered constant throughout; if now we consider e' to be variable, we may choose n and a so that the

constant (or rather the non-periodic) parts of u and of H^2 may have the same forms as before, and in this case we shall find the same relation between n and a as that which has just been found, and n will continue to signify the *actual mean motion* at the time to which θ belongs, but n and a will now become variable quantities, and, in order to satisfy our equations, it will be necessary to add certain periodic terms to u and H^2 which would not exist if e' were constant.

Suppose then that

$$u = \frac{1}{a}\left\{1 + \delta v - \frac{3}{2}m^2 e' \cos n't + m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) + \frac{7}{2}m^2 e' \cos(2\theta - 3n't) - \frac{1}{2}m^2 e' \cos(2\theta - n't)\right\},$$

and

$$H^2 = n^2 a^4\left\{1 + 2\delta\eta + \frac{171}{32}m^4 + \frac{2421}{64}m^4 e'^2 + \frac{3}{2}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) + \frac{21}{4}m^2 e' \cos(2\theta - 3n't) - \frac{3}{4}m^2 e' \cos(2\theta - n't)\right\}.$$

We will suppose e' to vary uniformly with the time, and very slowly, or, in other words, we will suppose

$$\frac{de'}{dt} \text{ to be constant, so that } \frac{d^2e'}{dt^2} = 0,$$

and we will neglect

$$\left(\frac{de'}{dt}\right)^2.$$

We must therefore recollect that $\frac{de'}{d\theta}$ is not constant, but is equal to

$$\frac{de'}{dt} \cdot \frac{dt}{d\theta} = \frac{1}{Hu^2} \cdot \frac{de'}{dt}$$

$$= \frac{de'}{ndt}\left\{1 + 3m^2 e' \cos n't - \frac{11}{4}m^2 \cos(2\theta - 2n't) - \frac{77}{8}m^2 e' \cos(2\theta - 3n't) + \frac{11}{8}m^2 e' \cos(2\theta - n't)\right\}.$$

5. In consequence of the variability of e', $\frac{du}{d\theta}$ will contain the additional terms

$$\frac{1}{Hu^2} \cdot \frac{1}{a}\left\{-\frac{da}{adt} - \frac{3}{2}m^2\frac{de'}{dt}\cos n't - 5m^2 e'\frac{de'}{dt}\cos(2\theta - 2n't) + \frac{7}{2}m^2\frac{de'}{dt}\cos(2\theta - 3n't) - \frac{1}{2}m^2\frac{de'}{dt}\cos(2\theta - n't)\right\} + \frac{1}{a} \cdot \frac{d \cdot \delta v}{d\theta},$$

or

$$\frac{1}{an}\left\{-\frac{da}{adt}-\frac{3}{2}m^2\frac{de'}{dt}\cos n't-5m^2e'\frac{de'}{dt}\cos(2\theta-2n't)+\frac{7}{2}m^2\frac{de'}{dt}\cos(2\theta-3n't)\right.$$
$$\left.-\frac{1}{2}m^2\frac{de'}{dt}\cos(2\theta-n't)\right\}$$
$$+\frac{1}{a}\cdot\frac{d\,.\,\delta v}{d\theta},$$

to the order of approximation required.

Therefore also $\frac{d^2u}{d\theta^2}$ will contain the additional terms

$$\frac{1}{an}\left\{10m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-7m^2\frac{de'}{dt}\sin(2\theta-3n't)+m^2\frac{de'}{dt}\sin(2\theta-n't)\right.$$
$$\left.+10m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-7m^2\frac{de'}{dt}\sin(2\theta-3n't)+m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}$$
$$+\frac{1}{a}\frac{d^2\,.\,\delta v}{d\theta^2}$$

neglecting

$$\frac{d^2a}{dt^2},\quad\left(\frac{da}{dt}\right)^2,\quad\text{and also}\quad m^3\frac{de'}{dt}$$

in the coefficients of the periodic terms.

Hence

$$\frac{d^2u}{d\theta^2}+u$$

contains the additional terms

$$\frac{1}{a}\left\{\frac{d^2\,.\,\delta v}{d\theta^2}+\delta v\right\}$$
$$+\frac{1}{an}\left\{20m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-14m^2\frac{de'}{dt}\sin(2\theta-3n't)+2m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}.$$

Also $\frac{\mu}{H^2}$ contains the additional term

$$\frac{\mu}{n^2a^4}\left[-2\delta\eta\right].$$

The other terms which enter into the first differential equation receive no additional terms of the order to which we restrict ourselves.

6. Also differentiating the expression for H^2, and including terms of the order

$$m^4e'\frac{de'}{dt}$$

in the non-periodic part, but only those of the orders

$$m^2\frac{de'}{dt}\quad\text{and}\quad m^2e'\frac{de'}{dt}$$

in the periodic part, we have the following additional terms in $\frac{d(H^2)}{d\theta}$, viz.

$$n^2a^4\frac{1}{Hu^2}\left\{\frac{2dn}{ndt}+\frac{4da}{adt}+\frac{2421}{32}m^4e'\frac{de'}{dt}-\frac{15}{2}m^2\frac{e'de'}{dt}\cos(2\theta-2n't)\right.$$
$$\left.+\frac{21}{4}m^2\frac{de'}{dt}\cos(2\theta-3n't)-\frac{3}{4}m^2\frac{de'}{dt}\cos(2\theta-n't)\right\}$$
$$+n^2a^4\left\{2\frac{d\,.\,\delta\eta}{d\theta}\right\}.$$

Also the right-hand side of the second differential equation contains the following additional quantity:—

$$m^2n^2a^4[4\delta v]\left\{3\sin(2\theta-2n't)+\frac{21}{2}e'\sin(2\theta-3n't)-\frac{3}{2}e'\sin(2\theta-n't)\right\},$$

which, as we shall immediately find, contains non-periodic terms of the order

$$m^4e'\frac{de'}{dt}.$$

Hence, taking the periodic parts of this equation, we have

$$2\frac{d\,.\,\delta\eta}{d\theta}=\frac{1}{n}\left\{\frac{15}{2}m^2e'\frac{de'}{dt}\cos(2\theta-2n't)-\frac{21}{4}m^2\frac{de'}{dt}\cos(2\theta-3n't)\right.$$
$$\left.+\frac{3}{4}m^2\frac{de'}{dt}\cos(2\theta-n't)\right\};$$

$$\therefore\quad 2(\delta\eta)=\frac{1}{n}\left\{\frac{15}{4}m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-\frac{21}{8}m^2\frac{de'}{dt}\sin(2\theta-3n't)\right.$$
$$\left.+\frac{3}{8}m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}.$$

7. Substitute this in the first equation, putting

$$\frac{\mu}{n^2a^3}=1$$

in the coefficients of the periodic terms, as these are only required to the order of m^2, and we obtain

$$\frac{d^2\,.\,\delta v}{d\theta^2}+\delta v=-\frac{1}{n}\left\{20m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-14m^2\frac{de'}{dt}\sin(2\theta-3n't)\right.$$
$$+2m^2\frac{de'}{dt}\sin(2\theta-n't)+\frac{15}{4}m^2e'\frac{de'}{dt}\sin(2\theta-2n't)$$
$$\left.-\frac{21}{8}m^2\frac{de'}{dt}\sin(2\theta-3n't)+\frac{3}{8}m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}$$
$$=-\frac{1}{n}\left\{\frac{95}{4}m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-\frac{133}{8}m^2\frac{de'}{dt}\sin(2\theta-3n't)\right.$$
$$\left.+\frac{19}{8}m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}.$$

$$\therefore\qquad \delta v=\frac{1}{n}\left\{\frac{95}{12}m^2e'\frac{de'}{dt}\sin(2\theta-2n't)-\frac{133}{24}m^2\frac{de'}{dt}\sin(2\theta-3n't)\right.$$
$$\left.+\frac{19}{24}m^2\frac{de'}{dt}\sin(2\theta-n't)\right\}.$$

Substitute this value of δv, and also the value of $\frac{1}{Hu^2}$, viz.

$$\frac{1}{n}\left\{1+3m^2e'\cos n't-\frac{11}{4}m^2\cos(2\theta-2n't)-\frac{77}{8}m^2e'\cos(2\theta-3n't)\right.$$
$$\left.+\frac{11}{8}m^2e'\cos(2\theta-n't)\right\},$$

for that quantity in the second differential equation, and equate the non-periodic parts which result from this substitution,

$$\therefore\ \frac{2dn}{ndt}+\frac{4da}{adt}+\frac{2421}{32}m^4e'\frac{de'}{dt}+\frac{165}{16}m^4e'\frac{de'}{dt}-\frac{1617}{64}m^4e'\frac{de'}{dt}-\frac{33}{64}m^4e'\frac{de'}{dt}$$
$$=\frac{95}{2}m^4e'\frac{de'}{dt}-\frac{931}{8}m^4e'\frac{de'}{dt}-\frac{19}{8}m^4e'\frac{de'}{dt},$$

or

$$\frac{2dn}{ndt}+4\frac{da}{adt}+\frac{963}{16}m^4e'\frac{de'}{dt}=-\frac{285}{4}m^4e'\frac{de'}{dt},$$
$$\therefore\quad \frac{dn}{ndt}+2\frac{da}{adt}=-\frac{2103}{32}m^4e'\frac{de'}{dt}.$$

8. The substitution of the values of δv and $\delta\eta$ in the first differential equation introduces no non-periodic terms depending on $\frac{de'}{dt}$; consequently the value of $\frac{\mu}{n^2a^3}$ remains of the same form as before.

Hence

$$\log\left(\frac{\mu}{n^2a^3}\right)=\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)+\frac{75}{32}m^4+\frac{1197}{64}m^4e'^2-\frac{1}{8}m^4(1+3e'^2$$
$$=\frac{1}{2}m^2\left(1+\frac{3}{2}e'^2\right)+\frac{71}{32}m^4+\frac{1173}{64}m^4e'^2;$$
$$\therefore\quad \frac{2dn}{ndt}+3\frac{da}{adt}=-\frac{3}{2}m^2e'\frac{de'}{dt}-\frac{1173}{32}m^4e'\frac{de'}{dt}-m^2\left(\frac{dm}{mdt}\right)$$
$$=-\left(\frac{3}{2}m^2+\frac{1173}{32}m^4\right)e'\frac{de'}{dt}+m^2\left(\frac{dn}{ndt}\right),$$

since

$$m=\frac{n'}{n},\quad\text{and}\quad\therefore\ \frac{dm}{mdt}=-\frac{dn}{ndt},$$

n' being constant.

Hence

$$(4-2m^2)\frac{dn}{ndt}+6\frac{da}{adt}=-\left(3m^2+\frac{1173}{16}m^4\right)e'\frac{de'}{dt},$$

also from above

$$3\frac{dn}{ndt}+6\frac{da}{adt}=-\frac{6309}{32}m^4e'\frac{de'}{dt};$$
$$\therefore\quad (1-2m^2)\frac{dn}{ndt}=-\left(3m^2-\frac{3963}{32}m^4\right)e'\frac{de}{dt},$$

and

$$\frac{dn}{ndt} = -\left(3m^2 - \frac{3771}{32}m^4\right)e'\frac{de'}{dt};$$

$$\therefore \quad 2\frac{da}{adt} = \left(3m^2 - \frac{3771}{32}m^4\right)c'\frac{de'}{dt} - \frac{2103}{32}m^4c'\frac{de'}{dt}$$

$$= \left(3m^2 - \frac{2937}{16}m^4\right)e'\frac{de'}{dt},$$

or

$$\frac{da}{adt} = \left(\frac{3}{2}m^2 - \frac{2937}{32}m^4\right)e'\frac{de'}{dt}.$$

9. These equations give the rate of variation of the quantities n and a. We will now show that n denotes the actual mean motion, as it did when e' was constant.

From the values of u and H^2 we find

$$\frac{dt}{d\theta} = \frac{1}{Hu^2} = \frac{1}{n}\left\{1 - 2\delta\upsilon - \delta\eta + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't)\right.$$
$$\left. - \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't)\right\},$$

or

$$\frac{ndt}{d\theta} = 1 + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't)$$
$$- \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't)$$
$$- \frac{425}{24}m^2e'\frac{de'}{ndt}\sin(2\theta - 2n't) + \frac{595}{48}m^2\frac{de'}{ndt}\sin(2\theta - 3n't)$$
$$- \frac{85}{48}m^2\frac{de'}{ndt}\sin(2\theta - n't).$$

Divide by

$$1 + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't$$

and take into account $m^4e'^2$ in the non-periodic term,

$$\therefore \frac{ndt}{d\theta}\left\{1 - 3m^2e'\cos nt\right\} = 1 - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't)$$
$$- \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't)$$
$$- \frac{425}{24}m^2e'\frac{de'}{ndt}\sin(2\theta - 2n't)$$
$$+ \frac{595}{48}m^2\frac{de'}{ndt}\sin(2\theta - 3n't) - \frac{85}{48}m^2\frac{de'}{ndt}\sin(2\theta - n't),$$

and therefore

$$\int ndt = \theta + 3me' \sin n't - \frac{11}{8} m^2 \left(1 - \frac{5}{2} e'^2\right) \sin(2\theta - 2n't)$$
$$- \frac{77}{16} m^2 e' \sin(2\theta - 3n't) + \frac{11}{16} m^2 e' \sin(2\theta - n't)$$
$$+ 3\frac{de'}{ndt} \cos n't + \frac{295}{24} m^2 e' \frac{de'}{ndt} \cos(2\theta - 2n't) - \frac{413}{48} m^2 \frac{de'}{ndt} \cos(2\theta - 3n't)$$
$$+ \frac{59}{48} m^2 \frac{de'}{ndt} \cos(2\theta - n't).$$

Hence θ differs from $\int ndt$ by periodic terms only, which proves the proposition.

The value of $\frac{dn}{ndt}$ above found agrees with that found in my paper published in the 'Philosophical Transactions' for 1853.

Note on the Constant of Lunar Parallax. By Professor J. C. Adams, M.A., F.R.S.

From the report of a discussion which took place at a late meeting of the Society, I have reason to believe that an explanation of the apparent discrepancy between the value of the constant of parallax given by me in the Appendix to the *Nautical Almanac* for 1856, and in the *Monthly Notices*, vol. xiii. p. 263, and the value of the constant found by Hansen in the Introduction to his Lunar Tables, may not be unacceptable to some of our members.

It will be proper to begin this explanation by recalling to mind that my formula, in the article of the *Monthly Notices* above referred to, does not represent the parallax itself, but rather the sine of that quantity converted into seconds of arc by dividing by sin 1″ or, which is the same thing, by multiplying by the number of seconds in the arc equal to the radius. The employment of the sine of the parallax instead of the parallax itself appears to be desirable both on theoretical as well as practical grounds.

In the first place, the sine of the parallax, being proportional to the reciprocal of the radius vector, is the quantity given directly by the lunar theory, and, in the next place, it is the same quantity which is wanted in the reduction of lunar observations.

What I have called the constant of parallax in the papers above referred to is, then, the constant term in the expression for the converted sine of the parallax, supposing the periodic

terms to be expressed in cosines of angles which increase in proportion to the time. The value found for this constant was $3422''.325$.

This quantity may also be called very appropriately the mean sine of the parallax, although I do not use the term in the papers referred to.

The value of the corresponding constant in the expression of the parallax itself is $0''.157$ greater than this, or $3422''.48$, which may appropriately be called the mean parallax.

The formula in the Introduction to Hansen's Lunar Tables does not give the sine of the parallax, but the *logarithm* of the sine of the parallax, and the constant which Hansen calls C is a quantity such that the constant term in his expression for the logarithm of sine of the parallax is

$$\log \sin C.$$

Now, it is plain that the constant term in the development of log sin parallax is a different quantity from the logarithm of the constant term of the sine of the parallax, and hence my constant of parallax differs from Hansen's quantity

$$\frac{\sin C}{\sin 1''}.$$

We may readily express the relation between these two constants in the case in which the orbit is supposed to be an undisturbed ellipse.

In this case, if the reciprocal of the radius vector, which is proportional to the sine of the parallax, be developed in terms of cosines of multiples of the mean anomaly,

then, a being the semi-axis major,

and e the eccentricity of the orbit,

the constant term in the development will be $\frac{1}{a}$.

In the same case, the constant term in the development of the logarithm of the reciprocal of the radius vector, expressed in terms of the same form as before, will be

$$\log \frac{1}{a}\left(1-\frac{1}{4}e^2\right)$$

very nearly, instead of $\log \frac{1}{a}$; so that if c denote the constant term in the former development, and $\log c'$ the constant term in the latter, we shall have

$$\frac{c'}{c} = 1-\frac{1}{4}e^2 \text{ very nearly.}$$

This relation will still be approximately though not exactly satisfied when the Moon's perturbations are taken into account.

Hansen himself, in a paper in the 17th volume of the *Astronomische Nachrichten*, p. 299, in which he gives the results which

he had obtained in a preliminary investigation of the lunar perturbations, finds that the number corresponding to the constant term in the logarithm of the sine of the parallax requires to be augmented by $2''{\cdot}71$ in order to reduce it to the constant term in the sine of the parallax itself.

Calling the parallax p, Hansen finds that the value of the constant term in $\log\left(\frac{\sin p}{\sin 1''}\right)$ is

$$\log(3419''{\cdot}35),$$

and hence he concludes that the constant term in $\left(\frac{\sin p}{\sin 1''}\right)$ is

$$3422''{\cdot}06.$$

By repeating Hansen's calculation and taking into account some small terms omitted by him, I find the amount of the reduction to be slightly less than the above, viz. $2''{\cdot}67$, so that the constant term in

$$\frac{\sin p}{\sin 1''}$$

according to Hansen's preliminary theory would be $3422''{\cdot}02$.

This value, however, is not immediately comparable with my own, being founded on different elements.

Both values are purely theoretical, depending on the ratio of the Moon's mass to that of the Earth, the ratio of the Earth's equatorial and polar axes, and the ratio of the Earth's radius to the length of the seconds' pendulum in a given latitude.

If M denote the mass of the Earth,

m that of the Moon,

A the Earth's equatorial radius,

R the Earth's radius at a point of which the sine of the latitude is $\frac{1}{\sqrt{3}}$,

P the length of the seconds' pendulum at the same point;

then the constant term of the sine of the horizontal parallax corresponding to the latitude just specified may be represented by

$$\left(\frac{M}{M+m}\cdot\frac{R}{P}\right)^{\frac{1}{3}}F,$$

and therefore the constant term of the sine of the equatorial horizontal parallax may be represented by

$$\frac{A}{R}\left(\frac{M}{M+m}\cdot\frac{R}{P}\right)^{\frac{1}{3}}F=\left(\frac{M}{M+m}\cdot\frac{A^3}{R^2P}\right)^{\frac{1}{3}}F,$$

where F is a factor which may be found by theory from elements which may be considered as known with all desirable accuracy.

The values of $\frac{M}{m}$, A, R and P employed in finding my constant are the following:—

$$\frac{M}{m} = 81{\cdot}5,$$

which corresponds very nearly to Dr. Peters' constant of Nutation;

A = 20923505 English feet
R = 20900320 „
P = 3·256989 „

R and P belong to a point the sine of the *geographical* latitude of which is $\frac{1}{\sqrt{3}}$.

A and R are the quantities found from Bessel's latest determination of the figure and dimensions of the Earth as given in *Astron. Nachr.* vol. xix., p. 216, supposing that

1 Toise = 6·394564 English feet.

P is found thus: according to the formula given in p. 94 of Baily's Report on Foster's Pendulum experiments, *Mem. of the R.A.S.* vol. vii., the square of the number of vibrations made in a mean solar day, at a point the sine of whose geographical latitude is $\frac{1}{\sqrt{3}}$, by a pendulum which vibrates seconds in London is

$$7441625711 + \frac{1}{3}(38286335) = 7454387823.$$

Also Captain Kater's determination of the length of the seconds' pendulum in London is

39·13929 inches = 3·2616075 feet.

Hence as the square of the number of vibrations made at a given place in a given time varies inversely as the length of the pendulum, we derive the value above given for P.

The values of the fundamental elements employed by Hansen are the following:—

$$\frac{M}{m} = 80$$

A = 6377157 metres
R_1 = 6370063 „
P_1 = 0·992666 „

and R_1 and P_1 belong to a point the sine of the *geocentric* latitude of which is $\frac{1}{\sqrt{3}}$.

The corresponding values of R and P for a point the sine of whose geographical latitude is $\frac{1}{\sqrt{3}}$ are the following:—

$$R = 6370126 \text{ metres}$$
$$P = 0{\cdot}992651 \quad ,,$$

And the constant term of the sine of the equatorial horizontal parallax may be represented either by

$$\left(\frac{M}{M+m}\,\frac{A^3}{R^2P}\right)^{\frac{1}{3}}F, \quad \text{or by} \quad \left(\frac{M}{M+m}\,\frac{A^3}{R_1^2P_1}\right)^{\frac{1}{3}}F_1.$$

In my calculation of the factor F, I took into account terms of the order of the square of the Earth's compression. It would otherwise have been useless to distinguish between R^2P and $R^2{}_1P_1$ or between F and F_1.

At the time when Hansen's paper appeared in the *Astron. Nachr.* Bessel's latest determination of the figure and dimensions of the Earth was not available. Hansen employed an earlier determination given by Bessel in *Astron. Nachr.* vol. xiv., p. 344, in which the results were affected by an error in the calculation of the French arc of the meridian which was discovered later.

Hence the corrections to be applied to the logarithms employed by Hansen in order to make them agree with those employed by me are the following, expressed in units of the 7th decimal:—

	Correction.
$\log\left(\frac{M}{M+m}\right)$	+987
$\log\left(\frac{A}{R}\right)$	+ 25
$\log\left(\frac{R}{P}\right)$	−150

The correction to be applied to Hansen's value of the logarithm of the constant term in the sine of the parallax is therefore

$$25+\tfrac{1}{3}(987-150) = 304 \quad \text{of the same units.}$$

And the corresponding correction of the constant term of the sine of the parallax will be $0''{\cdot}24$, and therefore according to Hansen's preliminary theory, employing my system of fundamental data, the value of this constant term will be $3422''{\cdot}26$.

In my independent transformation of Hansen's expression I found the rather more precise value $3422''{\cdot}264$.

This is less than my own value of the same constant by $0''{\cdot}06$ nearly, as stated in my paper in the Appendix to the *Nautical Almanac* for 1856.

I there intimated my belief that Hansen's definitive theory would probably be found to introduce a correction to his former value of the constant term in question, and this turns out to be the case.

In *Astron. Nachr.* vol. xvii., p. 298, the constant term in $-w$ which denotes the perturbations of the natural logarithm of the reciprocal of the radius vector, divided by sin 1″, is given as 1345″·281, but in the introduction to Hansen's Lunar Tables this same quantity is given as 1348″·840. Hence, the correction to the former value is 3″·559, and multiplying this by sin 1″ and by 3422″ we find the corresponding correction of the constant of parallax to be 0″·059, so that this constant becomes 3422″·323, a result which agrees perfectly with my own.

In this connection it may be worth mentioning that the only periodic term in which I found any difference much exceeding 0″·01 between my coefficients of parallax and those obtained by a transformation of the results of Hansen's preliminary theory was that which has the argument denoted by $t+z$ in Damoiseau's notation.

The corresponding term in $-w$ is in Hansen's preliminary theory

$$10''\cdot92 \cos (t+z),$$

whereas in the Introduction to the Lunar Tables this term is

$$8''\cdot73 \cos (t+z);$$

the correction to the coefficient is $-2''\cdot19$, and multiplying this as before by sin 1″ and by 3422″ we find the correction to the corresponding term of the sine of the parallax to be

$$-0''\cdot036 \cos (t+z),$$

and if this be applied to the value of this term in the preliminary theory, viz.

$$0''\cdot181 \cos (t+z),$$

the result is

$$0''\cdot145 \cos (t+z),$$

which agrees perfectly with my own.

It should be remarked that, in the Introduction to his Lunar Tables, Hansen still continues to use the same fundamental data as he had done in his earlier paper, so that the value of the constant term in the sine of the parallax according to the data adopted in the Tables is 3422″·08.

Note added June 17, 1880.

In Professor Newcomb's valuable transformation of Hansen's Lunar Theory which I have just received, it is wrongly assumed

that I employed the same data as Hansen for the figure and dimensions of the Earth, and that my value of P, viz. 3·256989 feet, relates like Hansen's to a point the sine of whose *geocentric* latitude is $\frac{1}{\sqrt{3}}$, whereas it should be the *geographical* latitude, as that is the latitude which enters into Baily's formula from which my value of P is deduced.

In consequence of this, Professor Newcomb finds a discrepancy of 0″·03 between Hansen's value of the constant of parallax and mine when both are derived from the same system of fundamental data; but it has been shown above that no such discrepancy exists.

By a typographical error, the value of P which Professor Newcomb quotes from me is printed as 3·256 89 feet, instead of 3·256989 feet.

Ephemeris for finding the Positions of the Satellite of Neptune, 1880–81. By A. Marth, Esq.

P, angle of position of the minor axis of the apparent orbit in the direction of superior conjunction.

a, *b*, major and minor semi-axis of the apparent orbit.

Long.= longitude of the satellite in its orbit, reckoned from the point which is in superior conjunction with the planet.

Gr. Noon.	P.	*a*.	*b*.	Log. *a*.	Log. *b*.	Long.	Diff.
1880.	°	″	″			°	°
Sept. 1	314·30	16·66	6·51	1·2216	0·8136	112·48	612·51
11	314·24	16·74	6·53	·2238	·8150	4·99	·46
21	314·14	16·81	6·54	·2257	·8156	257·45	·41
Oct. 1	314·01	16·88	6·54	1·2273	0·8156	149·86	·37
11	313·87	16·93	6·53	·2285	·8149	42·23	·34
21	313·71	16·96	6·51	·2294	·8135	294·57	·32
31	313·55	16·97	6·48	1·2298	0·8115	186·89	612·30
Nov. 10	313·38	16·97	6·44	·2297	·8089	79·19	·30
20	313·22	16·95	6·40	·2292	·8059	331·49	·31
30	313·07	16·91	6·35	1·2282	0 8025	223·80	·32
Dec. 10	312·93	16·86	6·29	·2268	·7989	116·12	·35
20	312·82	16·79	6·24	·2251	·7952	8·47	·38
30	312·74	16·71	6·19	1·2231	0·7917	260·85	612·43
1881.							
Jan. 9	312·68	16·63	6·14	·2208	·7884	153·28	·48
19	312·66	16·53	6·10	·2183	·7854	45·76	·53
29	312 67	16·44	6·06	1·2158	0·7828	298·29	·58
Febr. 8	312·72	16·34	6·04	·2133	·7808	190 87	·64
18	312·79	16·25	6·02	·2109	·7795	83·51	612·69
28	312·90	16·17	6·01	1·2088	0·7790	336·20	

These values are to be interpolated for the times for which the apparent places of the satellite are required, and the position-angles p and distances s are then found by

$$s \sin (P-p) = a \sin \text{long.}$$
$$s \cos (P-p) = b \cos \text{long.}$$

The satellite moves in the direction of decreasing position angles, and will be at its greatest elongations ("nf." in posit $P+90$ and distance a, "sf." in posit. $P-90$) and at its conjunctions ("sup." in posit. P and distance b, "inf." in posit. $P-180°$) at the following hours, Gr. M.T.—

"Sp." Elong.	h	"Inf." Conj.	h	"Nf." Elong.	h	"Sup." Conj.	h
1880.							
Aug. 31	15·2	Sept. 2	2·4	Sept. 3	13·7	Sept. 5	1·0
Sept. 6	12·2	7	23·5	9	10·8	10	22·0
12	9·3	13	20·6	15	7·8	16	19·1
18	6·4	19	17·7	21	4·9	22	16·2
24	3·5	25	14·7	27	2·0	28	13·3
30	0·5	Oct. 1	11·8	Oct. 2	23·1	Oct. 4	10·4
Oct. 5	21·6	7	8·9	8	20·2	10	7·4
11	18·7	13	6·0	14	17·3	16	4·5
17	15·8	19	3·1	20	14·4	22	1·6
23	12·9	25	0·2	26	11·5	27	22·7
29	10·0	30	21·3	Nov. 1	8·6	Nov. 2	19·9
Nov. 4	7·1	Nov. 5	18·4	7	5·7	8	17·0
10	4·2	11	15·5	13	2·8	14	14·1
16	1·3	17	12·6	18	23·9	20	11·2
21	22·5	23	9·7	24	21·0	26	8·3
27	19·6	29	6·8	30	18·1	Dec. 2	5·4
Dec. 3	16·7	Dec. 5	3·9	Dec. 6	15·2	8	2·5
9	13·8	11	1·0	12	12·3	13	23·6
15	10·9	16	22·1	18	9·4	19	20·7
21	8·0	22	19·2	24	6·5	25	17·8
27	5·0	28	16·3	30	3·6	31	14·9
1881.							
Jan. 2	2·1	Jan. 3	13·4	Jan. 5	0·7	Jan. 6	11·9
7	23·2	9	10·5	10	21·7	12	9·
13	20·3	15	7·5	16	18·8	18	6·1
19	17·3	21	4·6	22	15·9	24	3·1
5	14·4	27	1·7	28	12·9	30	0·2
1	11·4	Feb. 1	22·7	Feb. 3	10·0	Feb. 4	21·2

"Sp." Elong.		"Inf." Conj.		"Nf." Elong.		"Sup." Conj.	
1881.	h		h		h		h
Feb. 6	8·5	Feb. 7	19·7	Feb. 9	7·0	Feb. 10	18·3
12	5·5	13	16·8	15	4·0	16	15·3
18	2·5	19	13·8	21	1·1	22	12·3
23	23·6	25	10·8	26	22·1	28	9·3

Addition to the Ephemeris for Physical Observations of Jupiter.
By A. Marth, Esq.

Though the Ephemeris published on pp. 416-418 of the *Monthly Notices* contains the data necessary for the reductions of observations, it gives them only so far as they do not involve an assumed value of the planet's ellipticity, and supplementary computations are consequently needed.

Bessel has treated the questions connected with observations of an incompletely illuminated planetary disk in the sixth paper of his *Astron. Untersuchungen*, but he considers only observations made with a heliometer or double-image micrometer, so that some additions referring to observations made by means of webs are required.

If $e = \sin \varepsilon_0$ is the eccentricity of the planet's spheroidal surface or $\cos \varepsilon_0$ the proportion of the polar axis to the equatorial diameter, P the angle of position of the polar axis, B the latitude of the earth above the planet's equator, and $\sin \varepsilon = \sin \varepsilon_0 \cos B$, the outline of the apparent planetary disk will be an ellipse, the minor axis of which, in position-angle P, is $= 2a \cos \varepsilon$, if the major axis or the equatorial diameter is $= 2a$, and the radius of the disk in position-angle p will be

$$= a \cos \epsilon', \text{ if } \tan \epsilon' = \tan \epsilon \,.\, \cos (p - \mathrm{P}).$$

Provided that the limit of illumination passes through those points on the planet's surface which have the Sun's centre in their true horizon, the projection of the limit of illumination upon the disk will be an ellipse, the radius of which in position-angle p is $= a \cos \epsilon' \cos d'$, if

$$\tan \psi = \tan (p - \mathrm{P}) \cos \epsilon, \quad \text{and} \quad \tan d' = \tan d \cos (\psi - w),$$

where d and w have the same significance as in Bessel's paper.

The visible disk of the planet is generally composed of two half-ellipses, the direction of their common diameter or of the line of cusps being in position-angle

$$\mathrm{P} + w_1 \mp 90^\circ, \quad \text{where} \quad \tan w_1 = \tan w \cos \epsilon.$$

The limit of illumination is visible in the two quadrants adjoin-

ing the position-angle $P+w_1$, while the rim of the disk is visible in the two opposite quadrants adjoining the position-angle

$$P+w_1+180°.$$

A web or a line being moved to or from the centre of the disk in the direction of the position-angle p or $p+180°$ will be a tangent to the rim of the disk when its distance from the centre is

$$= a\sqrt{1-\sin^2\epsilon\cos^2(p-P)};$$

and it will be a tangent to the curve formed by the limit of illumination when its distance from the centre is

$$= a\sqrt{1-\sin^2\epsilon\cos^2(p-P)}\,.\,\sqrt{1-\sin^2 d\cos^2(p_1-w)},$$

where

$$\tan p_1 = \tan(p-P)\sec\epsilon.$$

Hence, by putting

$$\sin\nu = \sin\epsilon\cos(p-P) \quad \text{and}$$
$$\sin\delta = \sin d\cos(p_1-w),$$

the distance of the two tangents which touch the two curves and enclose the visible disk is found

$$= 2a\cos\nu\cos^2\frac{1}{2}\delta, \quad \text{or}$$

minus the defect of illumination, $2a\cos\nu\sin^2\frac{1}{2}\delta$,

$$= 2a\cos\nu.$$

In observations of right ascension and declination the values of p are

$$p = 90° \quad \text{and} \quad p = 0°.$$

Hence, by putting

$$\sin\nu = \sin\epsilon\sin P, \qquad \text{and} \quad \sin\nu' = \sin\epsilon\cos P,$$
$$\tan P' = \tan P\cos\epsilon, \qquad \tan P' = \tan P\sec\epsilon,$$
$$\sin\delta = \sin d\sin(P'+w), \qquad \sin\delta' = \sin d\cos(P'+w),$$

the differences of the limbs in right ascension and declination are found to be

$$\frac{2a\cos\nu}{15\cos D}\,.\,\cos^2\frac{1}{2}\delta \quad \text{and} \qquad 2a\cos\nu'\,.\,\cos^2\frac{1}{2}\delta'$$

(D being the planet's declination), or

$$\frac{2a\cos\nu}{15\cos D} \quad \text{and} \quad 2a\cos\nu'$$

when corrected for the defects of illumination

$$\frac{2a\cos\nu}{15\cos 3D}\,.\,\sin^2\frac{1}{2}\delta \quad \text{and} \quad 2a\cos\nu'\,.\,\sin^2\frac{1}{2}\delta'.$$

These latter values will be found further on in the table.

In case the declination-webs are not perpendicular upon the transit-webs, but are so inclined that the corresponding p is not $=0°$, but $=i$, the values

$$\sin\nu' = \sin\epsilon\cos(P-i)$$

and

$$\tan P'' = \tan(P-i)\sec\epsilon$$

must be substituted for those before given.

In measurements of the equatorial and polar diameters

$$p \text{ is } = P+90 \text{ and } = P.$$

Hence, if

$$\sin\delta = \sin d\sin w \quad \text{and}$$
$$\sin\delta' = \sin d\cos w,$$

the measured distances are

$$2a\cos^2\frac{1}{2}\delta \quad \text{and} \quad 2a\cos\epsilon\cos^2\frac{1}{2}\delta',$$

or the defect of illumination of the equatorial diameter

$$2a \text{ is } 2a\sin^2\frac{1}{2}\delta,$$

and that of the polar diameter

$$2a\cos\epsilon \text{ is } 2a\cos\epsilon\sin^2\frac{1}{2}\delta'.$$

If, in observing a planetary disk with a double-image micrometer, the images are separated in the direction p, the amount of separation which produces external contact of the two curves is, according to Bessel's investigation,

$$= 2a\sqrt{1-\sin^2\epsilon\cos^2\psi}\,.\left(1-\sin^2\frac{1}{2}d\cos^2(\psi-w)\right),$$

where

$$\tan\psi = \tan(p-P)\cos\epsilon.$$

Hence measurements in the two directions for which $\psi-w$ is $=0°$ and $=90°$, and which have been specially suggested, would give values of

$$2a\cos^2\frac{1}{2}d\,.\,\sqrt{1-\sin^2\epsilon\cos^2 w}$$

and of

$$2a\sqrt{1-\sin^2\epsilon\sin^2 w}.$$

The corresponding position-angles are

$$P+w' \quad \text{and} \quad P+w_1\pm 90^\circ,$$

where

$$\tan w' = \tan w \sec\epsilon, \quad \text{and} \quad \tan w_1 = \tan w\cos\epsilon,$$

and these angles would have to be known beforehand. The latter direction is that of the line of cusps.

Double-image measurements in the directions P+90 and P of the chief axes give values of

$$2a\left(1-\sin^2\frac{1}{2}d\sin^2 w\right)$$

and of

$$2a\cos\epsilon\left(1-\sin^2\frac{1}{2}d\cos^2 w\right)$$

and it appears preferable to take the measures in these directions.

In the following Ephemeris, which will require little further explanation, I give now some additional data for the proper reduction of observations. The defect of illumination of the polar diameter is insensible, and that of the equatorial diameter does not differ sensibly from the "greatest phase" given on page 416 ff.

The assumed value of *Jupiter's* equatorial diameter is still Bessel's $37''\cdot60$ at the distance $5\cdot20273$, corresponding to the semi-diameter $97''\cdot81$ at the distance 1. The assumed value of $\cos\epsilon_0$ or of the proportion of the polar axis to the equatorial diameter is $0\cdot9363$. It would be very desirable that these values should be substantially improved by modern observations; but, of course, only the best observations would serve the purpose.

The last column of the Ephemeris gives the elongations O − L of the point O of *Jupiter's* vernal equinox from the point L in the plane of his equator, which is in superior conjunction in reference to the Earth.

Greenw. Noon 1880		Polar Diam.	Difference between limbs. in A.R.	Difference between limbs. in Decl.	Defect of illumination. in A.R.	Defect of illumination. in Decl.	*d.*	*w.*	O−L.
June	18	$35''\cdot42$	$2''\cdot505$	$35''\cdot84$	prec. l.	south l.	$11^\circ\cdot25$	$271^\circ\cdot34$	$119^\circ\cdot638$
	23	35·94	2·543	36·36	$0''\cdot022$	$0''\cdot06$	11·49	271·42	118·939
	28	36·48	2·582	36·90	·023	·06	11·67	271·50	118·299
July	3	37·04	2·623	37·47	·024	·06	11·79	271·59	117·721
	8	37·62	2·665	38·06	·025	·06	11·85	271·68	117·208
	13	38·22	2·708	38·66	·025	·07	11·84	271·77	116·764
	18	38·84	2·753	39·28	·025	·06	11·75	271·88	116·391

Greenw. Noon		Polar Diam.	Difference between limbs. in A.R.	Difference between limbs. in Decl.	Defect of illumination. in A.R.	Defect of illumination. in Decl.	*d.*	*w.*	O—L.
1880.		"	"	"	"	"	°	°	°
July	23	39·47	2·798	39·92	·025	·06	11·59	271·99	116·092
	28	40·11	2·844	40·57	·024	·06	11·35	272·12	115·872
Aug.	2	40·76	2·890	41·22	·024	·06	11·04	272·26	115·732
	7	41·41	2 936	41·88	·022	·05	10·64	272·42	115·673
	12	42 05	2·982	42·53	·021	·05	10·16	272·61	115·697
	17	42·68	3·026	43·16	·019	·04	9·59	272·82	115·804
	22	43·28	3·068	43·77	·017	·04	8·95	273·08	115·993
	27	43·86	3·108	44·36	·014	·03	8·23	273·38	116·262
Sept.	1	44·40	3·145	44·90	·012	·03	7·42	273·74	116·607
	6	44·89	3·178	45·40	·009	·02	6·54	274·20	117·025
	11	45·32	3·208	45·84	·007	·01	5·60	274·84	117·510
	16	45·69	3 233	46·22	·005	·01	4·60	275·78	118·052
	21	45·99	3·252	46·52	·003	·00	3·56	277·14	118·643
	26	46·20	3·266	46·74	·001	·00	2·48	279·72	119·272
Oct.	1	46·33	3·274	46·88	·000	·00	1·39	286·39	119·929
	6	46·38	3·276	46·93	foll l.	north l.	0·42	329·70	120·601
	11	46·33	3·271	46·89	·000	·00	0·98	70·95	121·275
	16	46·20	3·260	46·75	·001	·01	2·07	82·24	121·937
	21	45·98	3·244	46·53	·002	·01	3·15	85·76	122·575
	26	45·67	3·222	46·23	·004	·01	4·21	87·47	123·177
	31	45·29	3·194	45·85	·005	·02	5·22	88·53	123·732
Nov.	5	44·85	3·162	45·40	·008	·03	6·17	89·25	124·229
	10	44·34	3·126	44·89	·010	·03	7·06	89·77	124·660
	15	43·78	3·086	44·33	·012	·04	7·87	90·17	125·017
	20	43·19	3·044	43·72	·015	·05	8·61	90·49	125·295
	25	42·56	3·000	43·09	·017	·05	9·27	90·76	125·491
	30	41·91	2·954	42·43	·018	·06	9·83	90·98	125·601
Dec.	5	41·25	2·907	41·76	·020	·06	10·31	91·17	125·624
	10	40·58	2·860	41·08	·021	·06	10·71	91·34	125·559
	15	39·91	2·813	40·40	·022	·07	11·02	91·48	125·408
	20	39·24	2·766	39·73	·023	·07	11·24	91·62	125·173
	25	38·59	2·721	39·07	·023	·07	11·38	91·74	124·856
	30	37·95	2·676	38·42	·023	·06	11·45	91·85	124·460
1881.									
Jan.	4	37·33	2·633	37·79	·023	·06	11·44	91·96	123·987
	9	36·74	2·592	37·19	·022	·06	11·35	92·05	123·443
	14	36·17	2·553	36·61	·021	·06	11·20	92·14	122·831
	19	35·62	2·515	36·05	·020	·05	10·98	92·22	122·155
	24	35·10	2·480	35·53	·019	·05	10·70	92·30	121·419

Greenw. Noon	Polar Diam.	Difference between limbs. in A.R.	Difference between limbs. in Decl.	Defect of illumination. in A.R.	Defect of illumination. in Decl.	*d.*	*w.*	O−L.
1881	″	″	″	″	″	°	°	°
Jan. 29	34·62	2·447	35·03	·017	·05	10·37	92·39	120·627
Feb. 3	34·16	2·416	34·56	·016	·04	9·99	92·48	119·783
8	33·73	2·387	34·12	·015	·04	9·55	92·57	118·892
13	33·33	2·360	33·71	·013	·03	9·07	92·67	117·957
18	32·96	2·336	33·34	·012	·03	8·56	92·77	116·982
23	32·62	2·314	32·99	·010	·02	8·01	92·89	115·971
28	32·31	2·294	32·67	0·009	0·02	7·42	93·02	114·927

The inclinations γ and the ascending nodes Γ of the orbits of the four satellites in reference to the plane of *Jupiter's* equator are the following, the nodes being reckoned from O, the point of the vernal equinox of *Jupiter's* northern hemisphere:—

	Sat. I.		Sat. II.		Sat. III.		Sat. IV.	
1880	γ_1	Γ_1	γ_2	Γ_2	γ_3	Γ_3	γ_4	Γ_4
	°	°	°	°	°	°	°	°
Feb. 14	0·0103	3·7	0·4654	13·20	0·1867	265·97	0·3272	329·50
April 14	·0105	1·8	·4665	11·21	·1858	265·64	·3277	329·56
June 13	·0106	0·0	·4676	9·23	·1849	265·31	·3281	329·64
Aug. 12	·0108	358·4	·4687	7·26	·1839	264·99	·3284	329·75
Oct. 11	·0109	356·9	·4697	5·30	·1827	264·66	·3286	329·86
Dec. 10	·0111	355·5	·4708	3·35	·1815	264·33	·3286	329·99
1881								
Feb. 8	·0112	354·2	·4718	1·40	·1802	263·98	·3286	330·13
April 9	0·0112	352·9	0·4727	359·46	0·1789	263·61	0·3284	330·28

If these values of Γ are added to the elongations O − L of the point O from superior conjunction, given in the last column of the preceding table, the angles $\Gamma + O - L$ are the longitudes of the ascending nodes of the orbits reckoned from that line in the plane of the equator which is in superior conjunction in reference to the Earth. Hence the latitudes of the satellites above the plane of *Jupiter's* equator are easily found, if their elongations are known.

I have received from Mr. Trouvelot some observations of passages of the red spot made in 1878. The time of the passage of the preceding end on Sept. 25 is only inferred from an estimate made at $6^h\ 30^m$ Camb. M.T.: "preceding extremity of red spot has passed central meridian 10 or 15 minutes." I retain it, however, in the additional list, and will now also not omit the result of Prof. Pritchett's own inference from his observations made on July 9, 1878, an account of which with an illustration is to be found in *The Observatory*, vol. ii., p. 308. Prof. Pritchett writes:—"I am very sure of the time of the transit of the centre of the spot over the Central Meridian on that day within 10 minutes—it must have occurred between $19^h\ 15^m$ and $19^h\ 25^m$ Gr. M. T."

1878.	Observed Passages. Prec. End. h m	Middle. h m	Foll. End. h m		Corresponding Longitudes. Rot. 870° 60. Prec. End. °	Middle. °	Foll. End. °	Rot. 870°·43. Prec. End. °	Middle. °	Foll. End. °
Pritchett, Glasgow, Missouri.										
July 9	between { 19 15	—	—	Gr.	—	245·6	—	—	324·5	—
	{ 19 25	—	—	,,	—	251·7	—	—	330·6	—
Trouvelot, Cambridge, Massachusetts.										
Sept. 25	{ 6 15	—	—	Cambridge	176·9	—	—	242·6	—	—
	{ 6 20	—	—	,,	179·9	—	—	245·6	—	—
30	—	6 35	—	,,	—	221·5	—	—	286·4	—
Oct. 2	—	7 45	—	,,	—	204·8	—	—	269·3	—
Nov. 12	—	—	6 50	,,	—	—	219·3	—	—	276·8
24	6 10	—	—	,,	199·7	—	—	255·2	—	—
29	5 15	—	—	,,	198·2	—	—	252·9	—	—
Dec. 16	—	6 35	—	,,	—	210·1	—	—	261·9	—
23	—	5 33	—	,,	—	217·6	—	—	268·2	—
28	—	4 45	—	,,	—	220·3	—	—	270·1	—

I can also add the first observation made by Prof. Pritchett since *Jupiter's* reappearance in the present season.

1880										
May 25	—	23 12	—	Gr.	—	306·6	—	—	268·8	—

Prof. Pritchett writes:—"The observation was made after sunrise, but was entirely satisfactory. The spot has changed very little since the autumn of last year."

It is to be hoped that proper observations have been made and may soon be forthcoming to clear up the early history of the spot in 1878.

An Examination of the Double-Star Measures of the Bedford Catalogue. By S. W. Burnham, Esq.

The micrometrical measures of double stars by Admiral Smyth, recorded in the "Bedford Catalogue," may, for the purpose of this examination be considered in two distinct classes, which are unlike in respect to the character of the measures and the order of stars observed. This classification is based wholly upon the fact whether or not other measures of these stars were made prior to 1844.

Class I. Double stars accurately measured by Struve, Sir William Herschel, South, and others, before the epoch of Smyth.

Class II. Double stars, and stars with distinct companions, which had not been carefully measured by any other observer up to the time of the publication of the "Cycle of Celestial Objects."

A cursory examination will show that the quality of the measures as to accuracy is very different. The measures of Class I. are, in the main, uniformly correct. This is shown by comparing them with earlier and later measures by other observers which are known to be correct. The only exceptions are in instances when the prior observations were palpably erroneous. In such cases the Cycle measures are equally in error.

The measures of Class II. are quite as uniformly either roughly approximate or grossly incorrect. The greater portion of the objects in this class are not double stars in any proper sense of the term, but are simply small stars in the same field with larger and generally prominent naked-eye stars. The distances are very great, and the objects themselves are devoid of any interest to a double-star observer. Some of the stars catalogued by Sir John Herschel which have been measured by Smyth belong to this class, as the distances were all estimated by Herschel, and the angles generally given from single readings. A few of the Struve stars, not measured at Dorpat, and rejected in *Mensuræ Micrometricæ*, are also thus inserted. The larger part of the objects in this class would be of easy measurement with a 5·9-inch aperture, and much easier than many in class I.

My attention was first called to the doubtful accuracy of the observations of the Bedford Catalogue by communications relating to the alleged disappearance of the companions to certain

stars, as, for instance, Procyon, 61 *Geminorum*, and others. The reliable catalogues and measures do not furnish a single instance of such changes, and it seemed improbable that these mysterious disappearances, resting entirely upon the unsupported evidence of a single observer, should be real. It appeared much easier to explain the alleged disappearances by assuming errors on the part of the observer, than by assuming the sudden extinction of suns and their possibly attendant worlds. A slight acquaintance with the Bedford Catalogue confirmed the correctness of this view. So many discrepancies were found, it seemed singular that so few of them had been noticed, and that so much time had been wasted in endeavouring to find stars which obviously never existed in the places assigned to them by Smyth. The accompanying observations furnish many additional and similar instances of the disappearance or displacement of companion stars.

It is not clear why most of the stars in Class II. were ever observed at all. In one place Smyth alludes to the determination of proper motion, but even for this purpose, whatever may be said of a careful and elaborate series of observations, a single micrometrical measure, even of the accuracy of ordinary observers, could be of but little value other than to aid in determining whether the principal star had a proper motion. As these stars are for the most part prominent naked-eye stars, the work for ascertaining proper motion has been done by meridian observations. By reason of the uninteresting character of the objects in Class II., very few of them have been measured, or have received any attention from observers since Smyth.

Several years ago I called attention to the character of the double-star measures of Smyth, but the subject attracted no general comment till the communication of Mr. Herbert Sadler was printed in *Monthly Notices*, R.A.S., for January, 1879. The subject matter of the discussion in the Society that followed is so fresh in the minds of the members that it is not necessary that I should allude to it further than to say that in response to urgent requests of several English observers and others I have made the series of observations of the Smyth stars which is herewith submitted.

In investigating the double-star measures of the Cycle, the first step is to examine the means and methods employed for the observations. Admiral Smyth used a refracting telescope of 5·9 inches' aperture, which he speaks of as being perhaps "the finest specimen of the late Mr. Tully's skill." Again he says:—"On repeated trials I find the instrument bears its highest magnifiers [22 to 1200] with remarkable distinctness, as is especially evinced by the roundness of small discs, the dark increase of vacancy between close double stars, and from particular portions of the Moon when dichotomised. I have, therefore, reason to presume that the curves of the lenses are in exact chromatic and spherical aberration throughout; and the focal distance accurately proportioned to the densities of the flint and crown glasses.

. . . . . Nothing can exceed the definition and sharpness with which the telescope comes to focus, even under low powers, and different sized apertures."

Certainly no higher tribute could be paid to the most perfect productions of the best modern optician. Indeed, few of them would warrant such an encomium, if the use of the "highest magnifiers" means that under any circumstances a power of 1200 on such an aperture would be practically an advantage. It is apparent that the accuracy of the work would not be necessarily affected by the size of the instrument when we recall the first-class measures of Dawes with 3·8 inches, and of Herschel and South with 3·75 inches, and other observers with apertures under 6 inches.

With regard to the driving-clock, Smyth says, "the performance is simple and effective," and as he nowhere charges any errors to this cause, we must presume that it was satisfactory.

The next, and perhaps the most important appliance of all in this connexion, is the micrometer. It is described as "a fine double-line position micrometer, whose screws have been rigidly examined under every revolution," and furnished with powers from 62 to 850. Again he says :—"My double-wired micrometer is one of the very best which the skill and practice of Mr. Dollond was able to produce, and is really a charming instrument to use."

The method of making the measures is described as follows :—"In measuring distances the alternate + and − reading of the micrometer was practised to get rid of the zero correction. The position, or angle made by the line joining the two stars with the direction of diurnal rotation at the meridian, was mostly taken by placing the stars between two dark parallel lines, and was altered many degrees backwards or forwards to prevent the eyes being biassed." He also says: —"It is proper to add that I seldom observed but on the finest nights, and that my objects, with rare exceptions, were limited to 30° South Declination, which from my observatory was well clear of vaporous addenda to the refraction."

The present investigation relates only to the stars in Class II. There is little to be said about the other class. As already stated, the Cycle measures of nearly all stars, difficult and otherwise, which had been previously measured, are in harmony with the earlier observations. A working list was made of all stars belonging to Class II. where both distances and angles are given by Smyth, and they have been observed from time to time during the past year. The results now found, together with the observations of Smyth, and other measures, where any have been made, are given in the following pages.

Perhaps comment on the results of this examination is not necessary, as the observations will speak for themselves.

The discrepancies are so great, and so constantly recurring that it is obviously impossible to explain them on the theory of the ordinary errors of observation. From the entire list of stars, rejecting a few where the identity of the particular companion is

uncertain, and those only partially measured by Smyth, we find from the 126 stars remaining that the mean difference between the present measures and those recorded in the Bedford Catalogue is 25″ in distance, and 7° in position-angle. That the measures of Smyth are but little, if any, better than mere eye-estimates, made without wires in the field, cannot be seriously doubted, to say nothing of the measures of stars which, from their occupying entirely different places, could not have been seen at all. All my early catalogues of new stars were published with estimated distances and angles, and I have given the result of a comparison of these estimates with the subsequent measures of Baron Dembowski, in the *Astronomical Register* for January 1876, from which it appears that the average error in the estimate of stars less than 1″ apart was 10°·2 in angle and 0″·11 in distance; and for stars separated from 1″ to 6″ the corresponding errors were 8°·4 in angle, and 0″·45 in distance. It will be remembered that not only were some of these stars exceedingly close, but nearly all of them difficult objects, and hard to see under any circumstances. It is evident that the angle of two distant stars could be more accurately estimated than that of a close pair. This comparison is perhaps not applicable to the probable error in estimating distances as large as those of the stars in Class II.; but I think it will be found that the estimated distances of wide pairs in the catalogue of Sir John Herschel are as near the truth as the distances in the Bedford Catalogue.

It is obvious that these observations are not micrometrical measures at all in the usual sense of the term, when used in reference to the work of all other observers from the time of Sir William Herschel to the present day. That such approximate results do not belong necessarily to the age in which they are made is apparent from the work of several contemporary observers using telescopes much inferior in point of light. The measures of Dawes with a 3·8-inch aperture, and of Herschel and South with $3\frac{3}{4}$-inch, will compare favourably with any modern work. It is true that many of Smyth's observations have "weight 1" attached to them; and in some cases that is described as being little or no better than an estimate; but it is also true that the same weight is given to a large number of measures in Class I., where the results are in harmony with other measures which are unquestionably accurate, and that these measures of the Cycle have been used for comparison and computation by all the authorities who dealt with the subject.

The intrinsic accuracy of the measures in Class II. cannot be inferred from the weight thus assigned. It is a noticable fact, however, that larger weights generally are attached to measures in Class I., although the stars are much more difficult, and to the measures of the components of a group which are not measured for the first time. For instance, in *β Lyræ*, to the measure of B is given "weight 9," while C and D, practically not more difficult and observed the same night, have each "weight 1." The result

is that all the Cycle observations, without reference to weight, have been uniformly regarded as measures, and not as mere estimates.

It could hardly be supposed that anyone would give the result of guesswork in tenths of a degree in angle, and tenths of a second in distance, in stars separated from 25″ to 200″ and more, however accurate his judgment might be.

It remains to seek for the most probable explanation of the extraordinary character of the "Cycle" measures. If those in Class I. were of the same character, as to apparent accuracy, as those in Class II., it would be difficult to suggest any explanation other than a general uncertainty about the whole work. That they were made in some way, and in good faith, will not now be and has not at any time been questioned by me.* We know that the observations in the Bedford Catalogue, which, so far as the double stars are concerned, could have been easily made in one year, are scattered over a series of years. It may, I think, be fairly assumed that they were made in leisure moments, without that care which a more zealous and experienced observer would bestow; with no definite idea of their publication and use; and as an amusement rather than as a serious astronomical work. If we assume that at the beginning the observer made it a practice, in measuring double stars, of setting the micrometer wires in accordance with the previous measure of other observers, for the purpose of identification, or for some other reason, and with the intention of making such changes in the wires as the appearance of the object seemed to warrant, we have at once a complete explanation of the very close agreement with other measures. Besides, this theory answers, as perhaps no other will, in accounting in the most satisfactory manner for the equally close agreement when the prior measures were widely in error. In such a star, for instance, as δ *Cancri*, where the companion would be a faint star in the Smyth telescope, it is very easy to see how, with the wire of the micrometer placed where the small star was expected to be found, the observer might imagine he saw the companion, and so record the position as a measure. There are many instances of large errors of this kind thus readily accounted for, which cannot be explained by the chapter of accidents, nor by any other theory of actual observation. Such a method of observing would hardly be expected to give accurate results in the measure of stars in Class II.

Before too severely condemning such course, it should be borne in mind that not only may the observations, as already suggested, have been made without any expectation of their future publication, and without any attempt at special and independent accuracy, but that the manner of observing may have

* It is due to Mr. Proctor, in this connection, to say that his explanation of certain statements under discussion respecting the Bedford Catalogue at the meeting of the Society before referred to was in exact accordance with the facts, and with the meaning of the writer.

been forgotten, or to some extent lost sight of, when the Bedford Catalogue was prepared.

With few exceptions, all the stars in the list prepared for this purpose have been observed, and the measures carefully made with the 18·5-inch refractor of the Dearborn Observatory. I have observed 148 stars, and made altogether more than 350 measures. The stars cited in the paper by Mr. Sadler before referred to were not put in the list except in a few cases where no recent measure had been made. In submitting these measures it is proper to say that generally they were made under unfavourable atmospheric conditions, and when double stars, properly speaking, could not have been observed. In another view it is important to remark the circumstances under which the observations were made.

It is not improbable that some of these stars may prove to be really double, and although no very close new components have been detected in the course of this work, they may hereafter be found, and perhaps with smaller apertures than that used here. Feeling that the measures had no scientific interest or value aside from their bearing upon the question at issue, I did not feel justified in devoting time to the work which could be more profitably spent in other directions. The present examination relates only to the micrometrical work of the Bedford Catalogue, and no attempt has been made to detect or correct errors in other directions. The magnitudes and colours of small stars are, to say the least, of very questionable accuracy in many instances.

In the preceding remarks I have assumed that my measures are substantially correct. These stars can be re-observed, and the accuracy of my work tested at any time, by anyone interested in the matter. There is no danger of these companion stars disappearing, and but little probability of any appreciable change except the trifling variation which, in a few instances, may result from proper motion.

Chicago,
1880, *February* 21.

MICROMETRICAL MEASURES.

β Cassiopeæ (p. 2).

Sm	P = 339°·6	D = 201″·0	1838·65
β	324·5	286·	1879·47
β	324·6	297·	1879·54

In this, and all other cases of single distances, only the nearest whole second is given. There are perhaps a dozen faint stars nearer the principal star than the one measured.

147 H. III. (p. 3).

Sm	P = 120°·0	D = 28″·0	1836·81	10 . . 11
β	142·9	63·53	1879·57	9 . . 10

There is a small star near B, 105° 12″.

γ *Pegasi* (p. 5).

Sm	P = 300°·2	D = 181″·0	1835·07	13
β	285·7	161·35	1879·47	10·5
β	285·5	162·29	1879·54	11·0

There is a small star near B, 199°·3 20″±.

ι *Ceti* (p. 6).

H	P = 14°·2	D = 45″ est.	1828	12
Sm	12·0	45·0	1838·82	15
β	16·5	61·89	1877·94	...
β	15·2	62·02	1879·87	11

12 *Ceti* (p. 8).

AB	H	P = 170° est.	D = 8″ est.	1825 ±
	Sm	180·8	6·5	1837·89
	De	185·2	8·66	1866·8
	β	186·0	8·63	1879·87
	β	187·2	8·76	1879·89
AC	Sm	113·4	201·0	1837·89
	β	110·4	211·84	1879·87
	β	110·2	212·98	1879·89

52 *Piscium* (p. 10).

H	P = 309°·6	D = 25″ est.	1828	14
Sm	311·0	25·0	1836·92	14
β	306·2	38·38	1879·54	11·3
β	305·8	38·14	1879·57	11·5

The companion is called "deep blue."

δ *Andromedæ* (p. 11).

Sm	P = 208°·3	D = 122″·0	1833·54	11½
β	194·4	228·63	1879·47	10
β	194·4	229·13	1879·54	10

α Cassiopeæ (p. 12).

	ℍ	P = 275°·4	D = 56″·17	1781·97	
	Sm	278·4	96·9	1831·86	
	Ja	279·8	61·19	1854·8	
	β	280 0	62·25	1877·57	
	β	279·8	62·51	1878 65	

P O. 146 (p. 14).

	Sm	P = 289°·9	D = 57″·9	1837·87	9
	β	289·8	65·12	1879·87	8·5
	β	290·0	65·07	1879·89	...

H 1046. (p. 17).

	H	P = 63°·8	D = 15′ est.	1825 ±	
	Sm	70·0	12·0	1835·68	
	β	68·0	16·38	1879·49	

σ^2 *Piscium* (p. 24).

AB	ℍ	P = 285°·5	D = 48″·13	1780·59	
	Sm	293·5	56·0	1832·81	
	β	293·5	55·79	1878·98	
	β	293·9	56·12	1879·57	
AC	Sm	235·0	140·0	1832·81	
	β	235·4	138·69	1878·98	
	β	233·4	138·14	1879·57	

This was examined by Mr. Challis with the Northumberland Equatoreal, but no measures are given with that instrument. Sm. has the extraordinary conclusion that 'an inference of binarity is deducible from a comparison of the registered epochs of ℍ and myself, at the rate of 0°·15 per annum, in a *nf* direction, indicating a highly elongated ellipse, with a period of upwards of 2000 years."

η Ceti (p. 27).

	Sm	P = 310°·7	D = 239″·10	1838·93	10
	β	304·5	224·86	1879·87	9·0
	β	304·5	225·59	1879·89	9·5

β Andromedæ (p. 27).

	Sm	P = 299°·0	D = 225″·0	1839·54	12

There is nothing in Sm.'s place. I have measured all the stars within 300″. If there is any choice, G or H probably come the nearest to the Cycle star, and agree best in magnitude. It might be, however, any one except B and C, which are too faint to have been seen with the Bedford glass.

AB	β	P = 269°·0	D = 84″·79	1878·82	12–13
		268·7	85·06	1879·56	12·5
AC		141·1	90·88	1878·82	11·5
		140·3	90·64	1879·56	12·0
AD		305·0	126·42	1879·49	10·5
		304·4	126·22	1879·56	11·2
		304·1	125·39	1879·58	11·0
AE		87·3	159·	1878·82	11·0
		87·3	157·66	1879·56	11·0
AF		207·7	210·	1878·82	11·0
AG		217·5	225·	1878·82	10·0
AH		293·6	305·	1878·82	10·5
		293·5	305·	1879·49	10·0
		294·0	304·	1879·55	...

Where only single distances were taken, the nearest second is given.

H 634 (p. 31).

H	P = 295° est.	D = 30″ est.	1825 ±	13–14
Sm	291·0	35·0	1836·89	14
β	271·6	35·44	1878·86	12
β	270·3	35·26	1879·87	11

The agreement would be of more consequence but for the statement of Smyth that his "angle and distance are mere estimations."

42 H VII. (p. 33).

Sm	P = 315°·0	D = 15″·0	1837·66	9 .. 10
β	324·5	13·31	1879·49	8·5 .. 8·5
β	324·5	13·80	1179·54	8·7 .. 8·7

40 *Cassiopeæ* (p. 37).

H	P = 241°·5	D = 45″ est.	1830	11–12
Sm	240·5	42·0	1834·95	12
β	236·9	53·08	1879·33	11
β	237·2	53·49	1879·49	10·8

76 *M. Persei* (p. 40).

Sm	P = 217°·0	D = 45″·0	1837·79	9 . . 14
β	229·6	33·80	1879·49	9·8 . . 10·5
β	228·7	33·76	1879·54	9·5 . . 10·0

The small star is described as "dusky."

107 *Piscium* (p. 41).

H	P = 316°·5	D = 60″ est.	1830	13
Sm	318·3	55·0	1837·03	14
β	340·9	77·95	1879·87	10
β	340·7	77·71	1879·89	10

H speaks of "a third 14–15 m." I have measured a 12 m. star nearer than this. This is not mentioned by Sm., and is probably too faint to have been seen with his aperture, as he speaks of the other as very minute.

β	P = 223°·2	D = 38″·45	1879·87
β	221·7	38·51	1880·01

ζ *Ceti* (p. 43).

Sm	P = 40°·4	D = 165″·0	1835·87	9
β	40·8	185·46	1879·87	9
β	40·6	185·88	1879·89	9·5

55 *Andromedæ* (p. 43).

H	P = 356°·5	D = 20″ est.		14
Sm	350·0	25·0	1832·95	16
β	355·1	60·00	1879·91	12·5
β	355·4	60·27	1879·84	11·5

I could not perceive any nebulous surrounding to the large star as noted by H. Sm. says the small star "was only caught by intense attention," and yet calls it "bluish."

α *Trianguli* (p. 44).

Sm	P = 179°·0	D = 110″·0	1834·67	11

There is no star in Sm.'s place. I have measured three distant stars from which to select the "Cycle" companion.

AB	P = 124°·1	D = 80″·62	1879·85	11·0
AC	182·3	227·90	1879·54	11·0
	182·3	228·	1879·85	11–12
		229·	1879·86	...
AD	172·8	307·77	1879·57	10·5
	172·8	309·	1879·85	...
	172·8	308·	1879·86	...

There is a 10·5 m. star in the direction of 160°·7. In identifying the Sm. star there is apparently but little choice. As a matter of fact, I have no doubt the star D is the one in question.

P I. 222. (p. 47).

AD	Sm	P = 359°·2	D = 165″·0	1834·99
	P. Sm	360·3	180·4	1862·74
	D [?]	361·6	182·5	1862·93
	Kn	360·8	183·68	1862·95
	β	360·7	183·86	1880·03

P I. 227. (p. 48).

	Sm	P = 113°·9	D = 43″·6	1834·87
	β	117·3	45·49	1879·87
	β	117·2	45·44	1879·89

χ *Persei* (p. 57).

AB	Sm	P = 354°·3	D = 65″·0	1835·82	12
	β	352·8	70·57	1879·54	10·5
	β	352·4	70·37	1879·56	10·5
AC	Sh	136·5	124·53	1824·99	
	Sm	43·2	122·0	1835·82	
	β	136·3	122·66	1879·54	

Sm. gives the angle of AC by Sh. as 43°·5, evidently mistaking *nf* for *sf*, from which the "Cycle" angle differs only 0°·3. This is not an error of 90°. The angle recorded by Sh. of 46°·5 *sf* is (90 + 46·5) = 136°·5, and 46°·5 *nf* is (90 − 46·5) = 43°·5. It is not easy to see how Sm. could make such an error in his own observation, as there is no such mistake found anywhere else in the work, and he expressly says, "the readings are reckoned from the north, in the direction *nfspn*, a method which Sir J. Herschel proposed for adoption in 1830, as well for its convenience as 'its avoidance of the continual and most annoying mistakes of the method introduced by his father, of reading by quadrants.'"

P II. 61 (=H 1123) (p. 66).

H	P = 252°·0	D = 15″ est.	1828–9
Sm	250·8	14·0	1837·78
β	248·5	20·00	1879·54
β	249·3	20·20	1880·05

θ Persei (p. 67).

Sm	P = 219°·0	D = 27″·0	1833·65
Fl	218·5	68·0	1877·63
β	218·5	68·87	1879·54

γ' Fornacis (p. 72).

H	P = 169°·4	D = 45″ est.	1830·
Sm	171·0	45·0	1837·94
β	157·5	49·15	1879·95

γ Persei (p. 75).

H	P = 224°·9	D = 60″ est.	1830·
Sm	226·0	55·0	1837·65
β	323·9	57·72	1879·54
β	323·6	57·72	1879·56

β Persei (p. 77).

The companions in order of distance are as follows :—

AB	β	P = 154°·8	D = 59″·37	1877·74	13
	β	155·8	58·21	1878·65	...
	β	155·2	59·59	1880·05	12·5
AC	β	144·2	67·83	1877·74	13+
	β	145·2	67·62	1878·65	...
	β	144·9	68·77	1880·05	11·8
AD	Sm	195·0	55·0	1835·63	11
	β	192·7	81·99	1877·7410	
	β	192·4	81·86	1878·65	...
	β	192·9	81·86	1880·05	10
DE	β	114·6	10·51	1877·74	12·5
	β	115·6	10·77	1878·65	...
	β	118·4	11·11	1880·05	12·5

52 *Arietis* (p. 77).

Sm	P = 85°·0	D = 105″·0	1835·88
β	82·9	101·56	1879·54
β	82·5	102·23	1879·89
β	82·5	102·82	1880·03

τ^4 *Eridani* (p. 79).

Sm	P = 240°·8	D = 150″·0	1836·90
β	236·0	160·45	1880·00
β	236·2	159·89	1880·02

There are four stars nearer than this, all readily seen with a 6-inch aperture.

AB	Cin⁴	P = 287°·1	D = 4″·77	1877·86
AC	β	99·3	39·97	1877·81
	β	100·0	39·99	1880·00
AD	β	293·3	123·08	1880·00
AE	β	275·9	130·05	1880·00

α *Persei* (p. 80).

Sm	P = 206°·0	D = 75″·0	1837·64	9
β	195·6	166·42	1879·54	11·0
β	195·4	165·68	1879·56	11·0

δ *Persei* (p. 82).

Sm	P = 315°·0	D = 140″·0	1833·74	11
β	313·3	97 20	1878·78	10·5
β	312·5	97·27	1879·54	10·0
β	312·0	97·86	1879·56	10·5

19 *Pleiadum* (p. 84).

H	P = 332°·8	D = 45″ est.	1831
Sm	335·0	45·0	1835·01
Rad	331·1	64·61	1863·1
β	329·3	66·52	1879·85
β	329·8	66·71	1879·86

The third measure is from *Radcliffe Observations*, vol. xxiii.

P. IV. 257 (p. 108).

AB	Sh	P = 304°.4	D = 38″.48	1822·09
	Sm	303·8	38·9	1831·95
	De	304·9	39·20	1874·4
	β	304·4	39·02	1880·06
AC	Sm	88·3	70·0	1831·95
	De	88·4	54·68	1874·4
	β	87·4	54·16	1880·00
	β	88·3	54·24	1880·06

The difference in the accuracy of the distances of the old and new stars is noticeable.

66 *Eridani* (p. 111).

Sm	P = 13°.8	D = 47″.0	1832·01	11
β	9·3	52·25	1879·91	9
β	9·4	52·74	1880·00	9·5

β *Eridani* (p. 111).

Sm	P = 147°.5	D = 120″.0	1830·98	12
β	143·9	116·60	1879·91	10·5
β	143·3	116·76	1880·00	10·8

H 3269 (p. 112).

H	P = 60°.4	D = 20″ est.	1831	
Sm	60·7	25·0	1837·73	
β	61·8	19·49	1880·00	
β	60·1	19·93	1880·07	

H 3272 (p. 118).

AC	H	P = 39°.3	D = 24″ est.	1831	7–8	13
	Sm	42·2	25·0	1836·71	7½	13
	β	42·8	33·31	1879·59	8	11·0
	β	42·7	33·65	1879·86	8	10·5

Two small stars nearer than this noted by H., all plain with 6-inch.

γ *Orionis* (p. 121).

Sm	P = 150°.0	D = 178″.0	1838·85
Challis	148·1	180·46	1841·19
β	143·4	178·49	1879·91
β	144·4	177·87	1880·00

In placing this star in Class II., the measure of 1841, quoted by Sm., was overlooked.

109 *P. Orionis.* (p. 124).

	H	P = 298°·1	D = 18″ est.	1830
	Sm	295·0	20·0	1834·71
	β	299·9	25·87	1879·99
	β	300·3	26·19	1880·00

β *Leporis* (p. 125).

	Sm	P = 67°·5	D = 210″·0	1832·00	11

There are two 10 m. stars in this quadrant as follows :—

AD	β	P = 75°·0	D = 206″·36	1879·88
AE	β	57·6	241·53	1879·88

ε *Orionis* (p. 134).

	Sm	P = 67°·9	D = 160″·0	1835·02
	β	56·8	180·01	1879·88
	β	57·3	179·84	1879·91

124 *Tauri* (p. 135).

BC	Σ	P = 315°·7	D = 5″·97	1830·5
	Sm	315·0	5·0	1835·65
	β	314·1	5·67	1879·85
AB	Sm	240·8	98·0	1835·65
	β	214·8	145·73	1879·85
AD	Sm	170·0	82·0	1835·65
	β	175·7	79·00	1876·85

The close stars BC make Σ 755.

ν *Aurigæ* (p. 139).

	H	P = 208°·2	D = 53″·71	1782·68
	Sm	201·9	85·0	1833·75
	β	207·0	54·97	1877·82
	β	206·7	54·32	1879·85

78 *M. Orionis* (p. 138).

	Sm	P = 32°·0	D = 45″·0	1836·79
		22·5	50·70	1878·10
	β	21·6	51·00	1878·16
	β	20·8	50·28	1880·02
	β	22·2	50·88	1880·03

B is double (= β 559).

37 *M. Aurigæ* (p. 140).

Sm	P = 357°·0	D = 25″·0	1836·79
β	345·8	17·45	1879·85

A coarse cluster, and many stars in the field. The two measured come nearest to the Cycle description.

β *Aurigæ* (p. 143).

☿	P = 35°·8	D = 169″·10	1782·23
Sm	38·2	185·0	1837·70
Ch	38·0	183·45	1841·96
β	39·0	183·72	1879·86
β	38·7	184·03	1880·05

The measure in 1841 by Challis is given in the Cycle. Hence this star properly belongs to Class I.

θ *Aurigæ* (p. 143).

☿	P = 286°·0	D = 35″ ±	1782·68
Sm	289·0	30·0	1832·64
OΣ	290·9	43·29	1852·12
Fl	293·3	45·50	1877·79
β	292·7	45·58	1880·05
β	294·4	45·44	1880·14

There is some change from proper motion. The distance of Sm. is much too small.

35 *Camelopardi* (p. 144).

H	P = 13°·8	D = 35″ est.	1830
Sm	14·4	30·0	1833·66
Ma	12·5	39·91	1843·3
De	13·1	39·44	1867·0

5 *Lyncis* (p. 148).

AB	H	P = 139°·5	D = 20″ est.	1830
	Sm	130·0	25·0	1833·77
	β	139·1	30·34	1879·86
AC	S	272·1	95·44	1825·05
	Sm	271·9	96·0	1833·77
	β	272·5	95·93	1879·86

μ Geminorum (p. 148).

Sm	P = 89°·0	D = 80″·0	1831·98
β	76·5	72·94	1880·00
β	76·7	72·38	1880·03

ζ Canis Majoris (p. 149).

Sm	P = 338°·0	D = 167″·0	1833·81
β	337·2	174·15	1879·88

β Canis Majoris (p. 150).

Sm	P = 339°·0	D = 104″·0	1833·76
β	339·0	184·63	1879·88
β	339·1	185·29	1879·91

10 *Monocerotis* (p. 151).

Sm	P = 225°·0	D = 7″·0	1832·99

There are two 10 m. stars in this quadrant. It is impossible to tell which Sm. observed, since his measures fit one about as well as the other.

AB	β	P = 256°·4	D = 76″·99	1879·88
	β	256·8	76·94	1879·91
AC	β	231·0	80·64	1879·88
	β	. . .	81·18	1879·91

γ Geminorum (p. 154).

AB	Sm	P = 335°·0	D = 75″·0	1830·80
	β	335·5	141·74	1880·03
AC	Sm	290·0	110·0	1830·80
	β	294·7	133·04	1880·03

There are several stars in the direction of B, but nothing nearer Smyth's place.

Sirius (p. 158).

Sm	P = 45°·0	D = 150″·0	1835·80

It has been long known that no star was to be found in or near this place. It presents an instance similar to that of Procyon, about which so much has been written, the only difference being that the error in the distance of the star near Procyon is much less than in this case. Mr. George Hunt has made a thorough examination of the vicinity of Sirius, and it is

quite sufficient to give his results. The real position of the Smyth 10 m. star is :—

Hunt	P = 47°·5	D = 10″·44 ±	1879·19

The distance, being too great for the screw, was roughly measured by the "comb" of the micrometer.

H 740 (p. 165).

H	P = 13° est.	D = 20″ est.	1825 ±
Sm	10·5	15·0	1835·22
Rad	7·2	20·95	1863·1
β	6·7	20·69	1880·00
β	8·0	20·91	1880·06

There is a very faint companion to B, 12·5 magnitude :—

β	P = 281°·9	D = 8″·67	1880·06

ζ *Geminorum* (p. 169).

AC	Sm	P = 85°·0	D = 65″·0	1831·81
	β	83·4	87·20	1880·00
	β	83·7	87·24	1880·03
AB	Sh	355·5	91·03	1821·23
	Sm	355·0	90·0	1831·81
	De	351·7	93·54	1874·06
	β	351·6	93·67	1880·00

δ *Canis Majoris* (p. 170).

Sm	P = 224°·0	D = 165″·0	1832·90
β	224·0	266·88	1879·98
β	224·1	265·97	1880·0

η *Canis Majoris* (p. 175).

Sm	P = 285°·0	D = 169″·0	1833·82
β	284·9	178·67	1879·98
β	285·0	178·63	1880·03

β *Canis Majoris* (p. 176).

AB	Sm	P = 80°·0	D = 35″·0	1831·84
	β	76·8	123·95	1879·98
	β	76·6	122·35	1880·00
AC	Sm	312·0	105·0	1831·84
	β	309·8	139·09	1879·98
	β	310·1	138·87	1880·00

There are several excessively faint stars on the following side of the primary, two of which were measured, the first being the nearest that could be seen.

	P = 82°·3	D = 47″·77	1880·16
	22·7	97·02	1879·98

The first is very faint, and will require probably a 12-inch aperture to show it, as it has less than half the light of the companion to Procyon.

H 2435 (p. 195).

H	P = 202°·2	D = 3″ est.	1830
Sm	205·0	4·0	1834·29
β	206·3	6·91	1880·03

P. VIII. 81 (p. 197).

Σ	P = 331°·5	D = 18″·20	1828·71
Sm	325·0	15·0	1838·69
β	331·1	17·95	1880·03

Observed because no other measures have been made since Struve.

θ Cancri (p. 198).

H I	...	D = 44″·87	1783·05
H	P = 61°·3	60 est.	1830
Sm	62·4	65·0	1833·31
Kn	59·3	58·41	1862·31
β	59·9	60·70	1879·25
β	59·5	60·72	1880·05

41 *Sextantis* (p. 234).

A and B	H	P = 305°±	D = 20″±		6 ... 17–1
	Sm	310·0	20·0	1838·31	16
	β	303·8	26·95	1878·10	
A and C	Sm	120·4	290·0	1838·31	10
	β	72·5	233·48	1879·33	9–10
	β	72·3	233·42	1879·34	

There is no star in the place given by Sm. of C. That measured by me is undoubtedly the one referred to in the Cycle.

β Ursæ Majoris (p. 237).

	Sm	P = 172°·6	D = 75″·0	1831·37	
	β	353·9	245·	1879·29	Single distance

There is no star in the place designated by Sm. as belonging to a "pale grey" 11 m. star. The one I have roughly measured is 9–10 m., and seemed most likely to be the one. There is a fainter star a little more than half the distance between the two.

δ Leonis (p. 244).

AB	Sm	P = 50°·0	ΔR.A. = 4ˢ·9	1836·21	
	β	44·5	D = 95″·20	1879·25	
	β	44·2	95·55	1879·28	
AC	Sm	345·9	ΔR.A. = 2ˢ·9	1836·21	
	β	345·2	...	1879·25	
	β	344·7	...	1879·28	

δ Crateris (p. 248).

	Sm	P = 94°·1	ΔR.A. = 19ˢ·8	1834·33	11
	β	107·8	...	1879·34	10·5

The difference in R.A. agrees substantially. There is a faint star between in the direction of 94°·7.

δ Ursæ Majoris (p. 262).

Sm	P = 127°·3	ΔR.A. = 20ˢ·4	1832·41	
β	124·1	D = 188″·59	1879·30	

β Corvi (p. 271).

Aβ	Sm	P = 119°·5	ΔR.A. = 27ˢ·4	1831·34	
	β	129·7	...	1879·25	
AC	Sm	306·5	ΔR.A. = 28·0	1831·34	
	β	292·0	...	1879·25	

8 *Can. Ven.* (p. 272).

Sm	P = 228°·0	D = 297″·0	1835·24	10
β	220·5	276·20	1879·26	9

δ Virginis (p. 286).

Sm	P = 144°·2	ΔR.A. = 5ˢ·8	1832·31	10·5
β	142·5	D = 151″·71	1879·25	10·5
β	142·2	152·36	1879·34	10·5

ε Virginis (p. 290).

Sm	P = 123°·8	D = 229″·0	1838·42	15
β	120·6	240·49	1879·25	
β	120·3	241·29	1879·34	11·8

53 *Virginis* (p. 292).

H	P = 30° est.	D = 50″ est.		
Sm	35·0	45·0	1833·40	15
β	9·3	70·80	1878·24	12–13
β	10·0	71·03	1879·25	12–13

Sm. called the small star "bluish." The positions of H are estimated from a diagram.

61 *Virginis* (p. 295).

H_J	P = 345°±	D = 73″·25	1783·0
Sm	340·6	ΔR.A. = 2ˢ·8	1832·31
Kn	22 6	D = 169″·29	1862·29
β	24·8	189·96	1878·35
β	25·8	191·43	1879·25
β	25·8	...	1879·28

There has been considerable change due to the proper motion of the large star, and there is a large error in the repetition of H_J's angle by Sm. Kn. has called attention to this, and states that Sm.'s angle should be about 14° or 15°.

α Virginis (p. 296).

Sm	P = 57°·3	ΔR.A. = 19ˢ·4	1833·28	10
β	32·9	D = 144″·82	1879·25	12
β	61·5	...	1879·34	10·5
β	61·9	359·84	1879·35	...

In the first observation a much nearer and fainter star was measured than the Sm. star. Later the proper companion was identified.

μ Hydræ (p. 300).

Sm	P = 80°·9	ΔR.A. = 19ˢ·5	1834·38
β	94·1	D = 138″·68	1879·28

72 *Virginis* (p. 301).

Σ	P = 16°·1	D = 30″·06	1831·53
Sm	18·5	25·0	1832·26
OΣ	16·8	28·98	1855·22
β	16·7	29·17	1879·25
β	15·6	29·30	1879·34

Sm. does not seem to have been aware that this was one of the Σ stars, and only refers to its being registered by $H_{\frac{5}{6}}$, but without measures. And yet the small star, which is only 11·5 of Σ's scale, was well enough seen to be described in the Cycle as "violet tint."

75 *Virginis* (p. 303).

H	P = 110°·3	D = 90′ ±	1830	13
Sm	112·0	93·0	1835·32	14
β	109·7	78·53	1879·25	11
β	110·5	78·04	1879·38	11·5

P XIII. 163 (p. 306).

Sm	P = 215°·0	D = 68″·0	1831·26	13
β	228·4	91·61	1879·24	11
β	227·9	91·56	1879·25	...

This faint star is called "ash-coloured" by Sm.

P XIII. 171 (p. 308).

Σ	P = 335°·7	D = 27″·76	1829·35
Sm	336·3	30·0	1830·99
β	335·4	27·93	1878·25
β	336·4	27·68	1878·28
β	334·7	27·51	1879·25

These are all the measures of this pair.

85 *Virginis* (p. 309).

H	P = 317°·8	D = 35″ est.	1830	15
Sm	320·0	30·0	1834·28	16
β	311·9	43·39	1879·28	
β	311·7	43·14	1879·37	11·7

η Bootis (p. 311).

Sh	P = 119·5°	D = 126·20″	1822·66
Sm	117·6	123·7	1832·42
β	110·8	114·75	1879·25
β	111·0	114·50	1879·34

The observations of Sh. were not noticed when this star and the next were entered on the list.

τ Virginis (p. 312).

Sh	P = 290°·0	D = 79″·29	1823·30	9
Sm	291·4	78·6	1831·24	8·5
β	289·6	79·57	1879·28	8·5

α Bootis (p. 315).

Sm	P = 49°·3	ΔR·A· = 15ˢ·1	1835·47	11
β	44·7		1879·25	...
β	44·8		1879·3	10

Very distant; called "lilac" by Sm.

P XIV. 95 (p. 322).

Sm	P = 266°·5	D = 40″·0	1836·49	12
β	268·4	25·54	1879·29	10
β	267·9	24·79	1879·32	10

This is described as "a truly difficult object," and best "seen on averting the eye," but called "bluish."

5 *Ursæ Minoris* (p. 323).

Sm	P = 135°·5	D = 45″·0	1833·61	11
β	129·4	56·45	1879·30	10·5

β Ursæ Minoris (p. 330).

Sm	P = 5°·5	D = 165″·0	1833·64	11
β	342·8	207·82	1879·30	10·5

δ Bootis (p. 338).

Sh	P = 79°·6	D = 105″·33	1822·80
Sm	75·0	110·0	1835·49
Σ	78·9	104·87	1835·60
β	78·7	104·54	1879·25
β	78·7	104·93	1879·34
β	78·4	104·93	1879·47
β	78·6	104·67	1879·48

ι Draconis (p. 344).

Sm	P = 46°·2	D = 117″·0	1833·60	9
β	50·0	254·67	1879·26	8·7

α Serpentis (p. 348).

H	P = 359°·5	D = 60″ est.		14
Lam.	179·7	61·48	1830 ±	
Sm	361·5	50·0	1837·32	15
β	354·5	58·89	1878·35	. . .
β	353·2	58·68	1879·30	12

The angle from Munich Observations should evidently be increased 180°.

ζ Ursæ Minoris (p. 351).

Sm	P = 210°·5	D = 310″·0	1834·64	11
β	212·9	313·48	1879·30	11·8

δ Ophiuchi (p. 356).

Sm	P = 131°·5	D = 319″·0	1834·45	10
β	127·2	265·70	1879·38	10
β	291·6	64·64	1879·38	12·5

I have measured another star much nearer than that observed by Sm.

υ Coronæ Borealis (p. 358).

AB	Sm	P = 24°·2	D = 86″·9	1831·59
	β	22·3	86·13	1879·29
	β	22·5	86·48	1879·34
AC	Sm	55·2	128·0	1831·59
	β	52·2	123·85	1879·29
	β	52·3	123·37	1879·34
BD	Sm	221·5	10·0	1831·59
	β	222·0	13·05	1879·29
	β	223·4	13·41	1879·34
Aa	β	29·9	56·60	1879·29
	β	29·2	55·37	1879·34

I have measured a faint star, 12 m. nearer the principal star than any before observed.

ν² Coronæ Borealis (p. 362).

	Sm	P = 16°·9	D = 137″·0	1838·57	12
	β	15·7	104·68	1879·29	
	β	15·5	104·45	1879·30	10

η Draconis (p. 365).

AF	Sm	P = 31°·0	D = 190″·0	1833·62	
	β	18·4	366·	1879·27	
	β	18·8	367·	1879·36	

There seems to be no doubt of the identity of the star. My measures are single distances only. Besides the close companion (OΣ 312), there are several faint stars nearer than the one observed by Sm., which are :—

AC	β	P = 83°·5	D = 107″·	1879·27	13
AD	β	158·3	. . .	1879·27	12–13
AE	β	172·1	. . .	1879·27	11

β Herculis (p. 366).

	Sm	P = 276°·0	D = 278″·0	1835·66	11*m*
	β	274·3	256·15	1879·30	9
	β	274·3	257·74	1879·34	9

η Herculis (p. 372).

Sm	P = 265°·0	D = 141″·0	1835·65	
W and S	261·4	113·7	1873·47	
W and S	261·6	114·3	1874·55	
β	261·6	113·39	1879·27	

At one time Σ suspected the large star to be a close pair, but afterwards rejected it as single. Of the correctness of this conclusion there has never been any doubt. Sm. measured the angle and distance in 1842.

η Ophiuchi (p. 379).

Sm	P = 260°·5	D = 269″·0	1833·61	13
β	267·4	256·24	1879·34	10·8
β	267·5	256·86	1879·41	10

36 *Ophiuchi* (p. 381).

Sm	P = 289°·9	D = 193″·8	1831·57
Ja	298·3	180·0	1854·07
Flm	306·4	199	. . .
β	305·9	198·03	1879·34

Flammarion gives the annual proper motion 1″·27 in the direction of 202°. The distance found by Jacob appears to be much too small. According to my observation, the distance in 1854 would have been about 192″, and Smyth's position substantially correct. The observation of Jacob is taken from the *Hand Book of Double Stars.*

ε *Ursæ Minoris* (p. 381).

Sm	P = 358°·5	D = 41″·0	1835·55	12
β	6·7	77·08	1879·27	11·5
β	6·4	78·23	1879·36	11·0

The companion called "pale blue."

λ *Herculis* (p. 390).

Sm	P = 295°·0	Δ R.A. = 23ˢ·4	1834·58	10
β	251·8	...	1879·30	10

Nothing in Sm.'s place. The one measured is probably the Cycle star, as it corresponds fairly in magnitude and difference of R.A. There are a good many faint stars nearer.

γ *Draconis* (p. 400).

Sm	P = 123°·5	Δ R.A. = 9ˢ·7	1832·63	12
β	116·3	D = 124″·77	1879·27	10·8

There are several stars nearer than this, measures of which will be found in my first series of observations (*Mem. R.A.S.* xliv.). A later observation of the nearest gives

P = 151°·8 D = 20″·90 1879·27 13

τ *Ophiuchi* (p. 402).

Sm	P = 114°·5	D = 83″·1	1832·55	10
Sm	115·0	82·7	1838·58	...
β	127·0	100·07	1879·34	...
β	126·8	100·53	1879·37	9·5

18 *M. Clyp. Sob.* (p. 415).

Sm	P = 322°0	D = 35″0	1835·56	9 .. 11
β	131·7	24·74	1879·37	8·7 .. 10·8
β	134·4	24·89	1879·43	9 .. 11

H 2002 (p. 414).

Sm	P = 250°0	D = 20″0	1836·55	8½ .. 10
β	253·6	19·57	1878·42	...
β	252·9	20·09	1879·37	10 .. 10

χ *Draconis* (p. 421).

Sm	P = 54°3	ΔR.A. = 14ˢ·6	1834·61	
β	26·1	D = 144″·22	1879·30	
β	26·9	145·00	1879·36	

There is a faint companion to the small star, BC, 315°±; 8″±.

2 *Aquilæ* (p. 426).

Sm	P = 133°·8	D = 55″0	1831·58
β	130·1	52·26	1879·34
β	130·4	52·52	1879·37

5 *Aquilæ* (p. 427).

Sm	P = 145°0	D = 30″0	1838·60
β	145·2	27·25	1879·34

25 *M. Sagittarii* (p. 420).

Sm	P = 149°5	D = 27″0	1836·60	8 .. 8

No stars answering this description in the cluster. The star which agrees most nearly in place and magnitude is Lalande 34207. The measures of this and a distant star are :—

β	P = 252°4	D = 66″·60	1879·34	8 .. 9·3

In the field, *sp* is a pair of 9 m. stars, 175° : 10″±.

In the field, *np*, are several 9–10 m. stars, about equal distances from each other. Two of them were measured :—

P = 301·3	D = 30·90	1879·34	9 .. 9·3

Probably the wide stars first observed are the ones mentioned by Sm.

110 *Herculis* (p. 428).

H	P = 108°·9	D = 60″ est.	1830
Sm	110·0	55·0	1831·68
β	92·3	61·16	1879·30
β	91·8	61·07	1879·57

There is a faint star between these, about one third the light of the old companion.

β	P = 95°·5	D = 44″·70	1879·30

P XVIII. 197 (p. 430).

Sm	P = 168°·9	D = 99″·0	1833·66
β	170·4	114·11	1879·34
β	170·6	113·86	1879·37

There is a 13 m. star near A, measured as follows:—

β	P = 356°·8	D = 22″·53	1879·37

ν^1 *Lyræ* (p. 432).

AB	Sm	P = 78°·0	D = 35″·0	1836·81
	β	70·2	36·46	1879·30
	β	70·8	36·01	1879·36
AC	S	124·0	59·84	1825·61
	Sm	124·1	58·6	1836·81
	β	122·1	58·65	1879·30
	β	122·4	58·51	1879·36
CD	Sm	208·0	12·0	1836·81
	β	212·9	18·23	1879·30
	β	212·3	17·52	1879·36

β *Lyræ* (p. 433).

AB	Σ	P = 149°·8	D = 45″·77	1835·2
	Sm	150·1	45·8	1834·73
	Hl	149·2	45·83	1876·7
	β	149·1	45·64	1878·39
AC	Sm	319·5	60·0	1834·73
	β	317·7	66·02	1879·30
	β	317·7	66·49	1879·36
AD	Sm	25·0	71·0	1834·73
	β	19·1	85·95	1879·30
	β	18·6	85·61	1879·36

It did not seem necessary to measure AB again, as the observations are sufficient to show the absence of any change. The measure by Hall is the mean result of 8 nights. This star is inserted here as an illustration of the apparent accuracy of the measures of Sm., when the star had been previously observed. The other companions, C and D, had not been measured at this time by other observers.

2024 *H. Aquilæ* (p. 437).

H	P = 220° est.	D = 10″ est.	1820+	11 . . 12
Sm	220·0	6·5	1835·57	7½ . . 11
Cin	219·5	11·28	1879·54	8·5 . . 10
β	218·8	11·31	1879·43	9·5 . . 10·8
β	218·6	11·23	1879·54	...

This is H 870, but there is an error of 5^m in the R.A. of H, and it is repeated by Sm. In another observation H gives magnitudes 9 . . 10.

γ *Lyræ* (p. 439).

Sm	P = 23°·5	Δ R.A. = 3ˢ·7	1834·59	11
β	20 9	D = 119″·44	1879·30	10·5

There are several stars nearer than this, besides the OΣ companion.

P XVIII. 299 (p. 440).

H	P = 270° est.	D = 50″ est.	1828	15
Sm	265·0	50·0	1838·55	14
β	275·9	42·30	1879·31	11·5
β	276·6	41·56	1879·35	11·5

There is a distant 9 m. star in 33°·1, and a smaller star, about two-thirds as far, in 15°·2. The principal star is 16 *Lyræ*.

ζ *Aquilæ* (p. 441).

Sm	P = 84°·0	Δ R.A. = 8ˢ·4	1832·47
β	76·9	D = 156″·65	1879·34

δ *Draconis* (p. 445).

Sm	P = 19°·7	Δ R.A. = 2ˢ·7	1834·78
β	27·8	D = 155″·45	1879·35
β	27·7	154·74	1879·66

There is a 12 m. star nearer than this :—

β	P = 354°·3	D = 90″·35	1879·35

δ *Aquilæ* (p. 446).

Sm	P = 259°·3	D = 96″·5	1833·62
P. Sm	275·6	94·96	1862·74
β	271·7	101·71	1879·34
β	272·0	100·96	1879·37

γ *Aquilæ* (p. 455).

Sm	P = 255°·0	D = 136″·0	1836·74
β	257·4	132·24	1879·31
β	258·0	132·93	1879·52

56 *Aquilæ* (p. 462).

Sm	P = 72°·5	D = 43″·0	1834·63	12
β	78·8	46·92	1879·51	11·8

β *Aquilæ* (p. 464).

Sm	P = 343°·0	D = 175″·0	1834·50	10
β	347·3	151·66	1879·34	10·5
β	347·2	151·77	1879·37	10·0
β	346·9	152·24	1879·43	10·5

α^2 *Capricorni* (p. 472).

Da	P = 220°·1	D = 44″·14	1830·60
Sm	221·8	43·0	1838·72
β	221·3	44·52	1879·43
β	221·2	44·09	1879·46
β	220·9	44·35	1879·58

Sm. does not mention the earlier measure.

α^2 *Capricorni* (p. 472).

Sm	P = 155°·7	D = 198″·0	1838·72	9
β	156·4	154·39	1879·43	9·5
β	156·1	154·47	1879·36	9·5

M. 29 *Cygni* (p. 478).

Sm	P = 299°·5	D = 55″·0	1835·48
β	295·1	66·27	1879·31

Not easy of identification. These stars are the nearest in position and distance to the observation of Sm., but differ in magnitudes. Sm. called the stars 8 and 11 m., and I have entered those measured by me, 8·5 and 8·8 m. There is a star, 11 m., nearer A than the one given above.

P = 319°·2 D = 38″·62 1879·31

1 *Aquarii* (p. 483).

		P	D		
AB	Sm	220°·0	20″·0	1838·69	13
	β	217·3	56·35	1879·43	11·5
	β	217·4	55·39	1879·46	11·5
AC	Sm	45·0	35·0	1838·69	14
	β	38·9	72·97	1879·43	11·3
	β	38·9	72·82	1876·46	11·3

α Delphini (p. 483).

		P	D		
	Sm	117°·8	105″·0	1835·72	13
	β	113·8	80·44	1879·31	10·8
	β	113·9	80·90	1879·37	. . .

Two other stars nearer A than this.

α Cygni (p. 485).

	P	D	
Sm	102°·5	108″·5	1837·65
Bond	88·6	95·5	1848·40
β	106·2	75·85	1879·31
β	105·8	75·41	1879·35
β	105·9	75·09	1879·40

There seems to be some error in the single observation by Bond. At my request a search was made for the original record at the Harvard College Observatory, but it could not be found.

η Cephei (p. 488).

	P	D		
Sm	29°·0	45″·0	1834·74	13
β	33·5	100·20	1879·35	11·5
β	34·1	100·89	1879·36	11·0

The companion is called "dusky."

χ Delphini (p. 490).

H	P = 22°·0	D = 6″ est.	1828	13
Sm	21·0	55·0	1838·84	14
β	21·5	40·03	1879·50	12
β	22·1	39·91	1879·51	. . .

The small star is said to be " pale lilac."

59 *Cygni* (p. 492).

AC	H	P = 143°·8	D = 20″ est.	1828
	Sm	142·0	30·0	1836·72
	β	140·6	26·73	1879·35

ζ *Cygni* (p. 497).

Sm	P = 51°·8	D = 105″·0	1835·55
β	67·2	101·39	1879·40
β	67·2	101·11	1879·41

α *Cephei* (p. 499).

Sm	P = 36°·0	D = 150″·0	1832·74	10
β	23·5	209·48	1879·35	10·8
β	23·4	208·95	1879·47	. . .

The small star is described as " pale sapphire."

β *Aquarii* (p. 502).

H	P = 322°·8	D = 20″ est.	1828	
Sm	320·0	25·0	1833·73	
β	320·0	34·26	1877·70	
β	318·0	34·60	1879·57	

ε *Pegasi* (p. 507).

AB	H	P = 322°·7	D = 90″·93	1782·89
	Sm	327·0	85·0	1833·67
	β	325·1	81·53	1879·51
	β	325·3	81·18	1879·57
AC	S	323·0	138·51	1825·15
	Sm	324·3	138·1	1833·67
	De	321·7	140·41	1874·8
	β	321·4	140·27	1879·51

The earlier measures are quoted by Sm.

P XXI. 312 (p. 509).

AB	H	P = 93° est.	D = 15″ est.	1825 ±	15
	Sm	95·0	15·0	1838·66	14
	P. Sm	92·5	20·32	1862·74	...
	β	92·7	20·71	1878·39	
	β	93·1	20·06	1879·54	11·0
AC	H	315 est.	20 est.	1825 ±	17
	P. Sm	323·6	21·74	1862·7	...
	β	324·0	24·40	1878·39	...
	β	325·8	24·10	1879·54	11·5

Sm., who could see B only, calls it "blue."

20 *Pegasi* (p. 509).

H	P = 32° est.	D = 40″ est.	1825·8	12
Sm	330·0	35·0	1838·66	14
β	326·1	51·28	1877·78	...
β	324·6	51·09	1879·51	11·5
β	325·9	50·90	1879·57	11·3

The small star was called "blue."

α *Aquarii* (510).

Sm	P = 44°·0	D = 116″·0	1837·71	13
β	41·5	115·08	1879·50	12·0
β	41·5	114·20	1879·51	11·5

π^1 *Pegasi* (p. 512).

AB	Sm	P = 258°·7	D = 74″·0	1835·56	
	β	261·7	72·62	1877·78	
	β	261·3	73·00	1879·40	
AC	Sm	90·2	185·00	1835·56	
	β	89·9	184·74	1879·40	

The agreement here is very singular, considering the distance of the components.

2 *Lacertæ* (p. 515).

Sm	P = 6°·9	D = 35″·0	1835·84	13
β	9·5	48·44	1879·46	11·0
β	9·9	47·96	1879·49	10·7

ζ Pegasi (520).

	Sm	P = 15°·5	D = 65″·0	1831·68	13
Aa	β	137·8	64·33	1879·54	11
Ab	β	5·2	177·	1879·51	11

Nothing in the place of Sm. The more distant of the two I have measured is probably the one. Single distance only of that star.

ξ Pegasi (p. 522).

AB	H	P = 122°·8	D = 15″ est.	1828·	18
	Sm	120·0	15·0	1834·79	15
	De	117·7	12·17	1866·8	12·0
	β	112·0	11·99	1878·62	...
	β	111·4	11·87	1878·63	...
	β	113·0	11·99	1879·54	11·8
AC	Sm	32·5	110·0	1834·79	12
	β	21·8	127·03	1879·51	11·5
	β	21·7	127·63	1879·54	11·0

β Pegasi (p. 527).

H	P = 204°·1	D = 80″ est.	1828·	16–17
Sm	199·0	75·0	1836·88	15·0
β	208·0	97·91	1879·49	11·5
β	208·5	98·86	1879·53	11·0

4 *Cassiopeæ* (p. 534).

AB	Sm	P = 225°·0	D = 97″·5	1834·67
	β	226·2	98·13	1879·47
AC	Sm	260·0	218·0	1834·67
	β	258·3	214·35	1879·47
CD	Sm	256·0	10·0	1834·67
	β	35·3	10·30	1879·47

The comparatively close agreement of the measures, with the exception of the angle of CD, is remarkable, if there are no prior measures. I am not aware of any, but Sm. says :—"When the great Northumberland Equatoreal was mounted at Cambridge, I requested the Rev. James Challis to scrutinize the object, which he kindly did, with similar results to my own."

ι *Piscium* (p. 536).

Sm	P = 39°·5	D = 199″·0	1835·83
β	33·5	281·	1879·54

Single distance only. There is a very faint star, 13–14 m., much nearer, but too faint to be seen with any moderate aperture, as it would be decidedly fainter than H's 20 m., 288°·2 : 54″.

κ Andromedæ (p. 536).

AB	Sm	P = 186°·0	D = 47″·0	1836·72	14
	β	188·9	46·34	1878·78	11
	β	189·1	47·16	1879·46	11
	β	188·1	46·43	1879·41	11
AC	Sm	295·5	98·5	1836·72	12
	β	294·6	103·17	1879·46	11

The small stars are called "dusky" and "ash-coloured."

P. XXIII. 171 (p. 537).

AB	Σ	P = 103°·8	D = 5″·38	1831·85
	Sm	105·0	5·0	1833·75
	β	98·7	5·58	1879·46
AC	Sm	155·0	175·0	1833·75
	β	129·9	381·	1879·46

Σ did not measure the third star. The difference in accuracy in the measures of Sm. of these two stars is very apparent, and yet, according to his estimate, the distant star is three magnitudes brighter than Σ's companion.

Note.—Since the foregoing observations were forwarded, I have measured two of the few remaining stars of Class II. This substantially concludes the work. The only wide pair not measured is 44 M. *Cancri* (p. 200).

ψ *Leonis* (p. 220).

Sm	P = 140°·0	D = 260″·0	1834·19
β	139·7	280·38	1880·15
β	139·5	280·66	1880·17
β	139·7	279·73	1880·18

30 *Canis Majoris* (p. 174).

Sm	P = 73°·0	D = 85″·0	1834·83
β	77·9	84·40	1880·19
β	78·0	84·46	1880·21

Although the two near companions of H are easily seen with

6-inch, Sm. does not mention them. I measured them as follows :—

A and B	P = 91°·5	D = 7″·72	1880·19
	90.4	7·97	1880·21
A and C	81·0	14·30	1880·19
	79·6	14·35	1880·21

Notes on a Paper entitled "An Examination of the Double-Star Measures of the Bedford Catalogue," by S. W. Burnham, Esq. By E. B. Knobel, Esq.

The above paper by Mr. Burnham is mainly devoted to an examination of those objects in the Bedford Catalogue designated by the late Admiral Smyth as "stars with comites," of wide double stars, and of stars with faint companions.

Most observers who have bestowed any attention on this class of so-called "double-stars" have noticed discrepancies, in their results, sometimes of considerable amount, compared with the figures given in the "Cycle." My attention was in the first instance drawn to discrepancies in some of the Cycle results, about the year 1867 or 1868, by the Rev. E. Lowe, of Atherstone, who was then engaged in a systematic examination of all the objects in the Bedford Catalogue, with the fine $8\frac{1}{4}$-inch Alvan Clark Refractor, originally belonging to Mr. Dawes, and which now forms the equatorial telescope of the Temple Observatory at Rugby.* Six years ago I measured several of these objects, and finding that my results differed widely from those in the Cycle, I communicated them to a then member of the council of the Royal Astronomical Society, stating that "I could not but think that Smyth's measures were in error." I was strongly counselled to publish these results, but I did not think them sufficiently numerous to be of much value.

Mr. Burnham's micrometrical measurements are very valuable as correcting the, in many cases, erroneous figures given in the Cycle for the position-angle and distance of these wide objects.

But there are many points connected with Admiral Smyth's observations of these stars which have either escaped Mr. Burnham's notice, or which are contained in publications and MSS. to which he may not have had access.

It is eminently desirable that in investigations of this kind every possible source of information should be carefully consulted, and that, in order to explain discrepancies, we should

* If my memory serves me right, Mr. Lowe's results were inserted in pencil in the margin of his copy of the Bedford catalogue which, I believe, is now at the Rugby Observatory.

endeavour to seek for a clue to them in the author's own writings, or in his known mode and means of observation, rather than in a hypothesis which may not be substantiated by anything therein contained.

The classification by Mr. Burnham of the stars into two groups, of 1. "Double stars accurately measured by Struve, Sir William Herschel, South, and others, before the epoch of Smyth;" and 2. "Double stars and stars with distant companions, which had not been carefully measured by any other observer up to the time of the publication of the Cycle," is more arbitrary than natural, and rather prejudges the whole question, especially as so little evidence is offered in support of the statements with regard to Class I.

The objects in Class II. consist almost entirely of stars with distant or faint companions which were inserted in the Bedford Catalogue for various reasons: either the star had a history, or it had been observed for a special purpose, or it was near something else, or it possessed some peculiarity in name which afforded the author an opportunity of discussing the oriental history of it, or it was a Greenwich star, or a companion was given to the star as a means of future identification, or "the angle and distance of an eligible companion was marked for the purpose of watching the motions of the larger component with an expected heliometer; as this was but a preparation, they are only registered on the lowest weight." *

The objects selected by Mr. Burnham and placed in Class II. when examined with reference to the printed Bedford Catalogue and the MSS. in Admiral Smyth's writing belonging to the Royal Astronomical Society, may be thus summarised:—

They comprise 150 of the celestial objects in the Cycle.

Of these, 79 are titled by Admiral Smyth "stars and comites;" 80 of the objects do not appear to have been micrometrically examined at all, the distance of the components is stated to have been simply estimated, or determined by Δ A.R. (equivalent to an estimation).† In this connection it is most important to take into consideration the weights assigned by Admiral Smyth to the observations discussed by Mr. Burnham. The 150 selected objects comprise 180 determinations of the position and distance of the components. Of these 180 distances, 148 have weight 1 assigned to them, 21 have weight 2, and 11 have higher weights. For the position-angles, 96 have weight 1, 52 have weight 2, and 32 higher weights.

As 82 per cent. of the distances have weight 1 assigned to them, we have first to enquire what value was attached by the

* *Cycle*, vol. i. p. 430.

† "Distance is a mere estimation by Δ A.R. over a small bar in the eyepiece expressly fitted for such cases."—*Cycle*, vol. ii. p. 462.

"The position and distance of the nearest of these was obtained by estimations made in a dark field for the angle and the Δ A.R. by a bar for the distance."—*Cycle*, p. 446.

author to such a weight. The answer is sufficiently clear. In Cycle, vol. i. p. 426, Admiral Smyth states that he has subjoined to the measurements of position and distance "weights of the comparative value of the observations in numbers from one to ten, *the first representing nearly worthlessness*, and the latter perfection." The paragraph already quoted * shows a class of observations to which weight 1 was attached. In vol. i. p. 428 Admiral Smyth writes, "The extremely faint objects were *estimated* either by an annular micrometer or a vertical bar which I had fitted to an excellent eyepiece with a small hole in its centre." Many of these extremely faint objects are included in Mr. Burnham's selections, and they have all weight 1 attached to them. In "Speculum Hartwellianum," p. 229, he says—"In the Cycle there is a slight typographical error in the distance of C—namely, instead of 15″·1 it should have been 15″·0, *as the trifling sum of weight* (W. 1) *would indicate.*" And at p. 260 he says:—"The distances are *mere guesses to which my lowest weights* (W. 1) *are applied*, but perhaps 0 ought to have been introduced into my scale." Should any further doubt exist as to what value Admiral Smyth attached to an observation with weight 1 appended, it is certainly decided in vol. ii. p. 453, where he says, "And it should be remembered that such mere points of light were scrutinised to prove the power of the instrument for its general work, rather than to establish data for an epoch by estimation only. *Thus the mark* (*w* 1) *serves only to show that the object was identified.*" † These words are most clear and explicit, and though it is undoubtedly and unfortunately true that subsequent observers have taken every figure in the Cycle as the result of careful micrometrical measurement, they have done so without considering, and in spite of, the author's distinct statements to the contrary.

It remains to consider how it happens that whereas 82 per cent. of the Distances have W. 1 attached, only 53 per cent. of the position angles have that low weight.

The micrometrical means of Admiral Smyth consisted not only of the Parallel Wire Micrometer described by Mr. Burnham, but also of what are more pertinent to the observations under discussion, of the Annular Micrometer for estimating distances of faint objects,‡ the bar micrometer "expressly fitted" for estimating distances by Δ A.R.§ and the Spherical Crystal Micrometer, which he found "excellent for ascertaining the position of objects too faint for illumination." ‖ The annular micrometer consisted of a steel ring inserted in a disc of glass, the radius of the ring being 472″·5 seconds of arc.¶ The Spherical Crystal Micrometer was a very favourite instrument with Admiral Smyth, and the principles of its construction are described

* *Cycle*, vol. i. p. 430.

† For a further explanation of this value, see notes to η Herculis appended.

‡ *Cycle*, vol. i. p. 428.

§ *Cycle*, vol. ii. pp. 446, 462.

‖ *Cycle*, vol. i. pp. 339, 428.

¶ *Cycle*, vol. i. pp. 339, 385.

in the references above given. His original instrument is in the possession of the Astronomer Royal, who has most kindly lent it to me for the purpose of this enquiry, and to endeavour to throw light upon the following quotations from the Smyth MSS. of the Cycle belonging to the Royal Astronomical Society :—

ζ Ceti. "By spherical crystal. Pos. 139° 24′ from 180° = 40° 26′" (this is a clerical error for 40° 36′).

δ Persei. "By rock crystal. Pos. 224° 40′ − 90° = 315° 0′" (this evidently should be + 90°).

δ Canis Majoris. "By spherical crystal = 314°·0′ − 90° = 224° 0′."

61 Virginis. "Crystal at 160° 35′
180
340° 35′" (180° + 160° 35′).

1 Piscium. "Pos. = 140° 30′ = 39° 30′" (evidently 180° − 140° 30′).

These quotations indicate that the reading had to be added to or subtracted from 90° or 180°, and hence a fruitful source of error.

For the range of this micrometer I find that the spherical crystal revolves through an arc having 45 divisions on each side of zero. The value of one division according to Admiral Smyth (Cycle, vol. i. p. 339) = 0″·27, consequently 12″·15 was the greatest possible distance that could be measured with it on his telescope.

A toothed circle graduated from 0° to 360° is fitted to the micrometer, round which the spherical crystal can be made to revolve. Against the 360° graduation there is a small projecting piece of brass, which evidently served for the purpose of inserting the instrument in the draw tube of the telescope in a definite position with regard to the meridian. It is apparently very doubtful how accurately this circle could be adjusted for measuring position-angles, and it is probable, from the absence of any wire or fiducial line for determining accurately the north and south points of the field, that such adjustment was uncertain.

As this micrometer afforded Admiral Smyth a far more reliable instrument for measuring the position-angles of faint objects than he had for their distances, so we find that the average weight he has given to such position-angles are higher than to distances.

I have investigated in a brief manner, but as carefully as the limited time at my disposal would allow, the employment of the Spherical Crystal Micrometer for determining position-angles. For this purpose I have arranged some artificial wide double stars at definite position-angles, and at distances ranging from 10 to 210 seconds of arc. The position-angles were measured exactly after Admiral Smyth's method, by pointing the double image of A in the direction of B.

As the axis of the spherical crystal can be revolved freely

round the circle in either direction without any stop, the result is that we may obtain *two* readings for every position-angle.

The following table of the micrometer readings corresponding to a series of position-angles may be of interest.

Position-Angles.		Micrometer Readings.	Position-Angles.		Micrometer Readings.
0° or 360°	=	360° or 180°	180°	=	180° or 0°
15	=	345 „ 165	195	=	165 „ 345
30	=	330 „ 150	210	=	150 „ 330
45	=	315 „ 135	225	=	135 „ 315
60	=	300 „ 120	240	=	120 „ 300
75	=	285 „ 105	255	=	105 „ 285
90	=	270 „ 90	270	=	90 „ 270
105	=	255 „ 75	285	=	75 „ 255
120	=	240 „ 60	300	=	60 „ 240
135	=	225 „ 45	315	=	45 „ 225
150	=	210 „ 30	330	=	30 „ 210
165	=	195 „ 15	345	=	15 „ 195

From this we gather the following mode of reducing the readings. Stars in north following and south following quadrants—

Position-angle = 360° − mic. reading,
or 180° − mic. reading.

Stars in north preceding or south preceding quadrants—

Position-angle = 360° − mic. reading,
or 180° + (360° − mic. reading).

The first thing that strikes one is the very great liability to error in observations of positions with this instrument, and knowing how freely it was used by Admiral Smyth, it is enough from distinct instances and inferentially to account for a large number of discrepancies. The quotation already given with regard to ζ Ceti reveals a clerical error in the reduction. With δ Persei the correction to the reading is given as −90° instead of +90°. As this position is *np*, Admiral Smyth's correction of +90° is in error, though taking the reading as 225° the result is the same.

δ Canis Majoris. "By spherical crystal = 314°·0 − 90° = 224°·0."

This is clearly in error. Reading 314° for a star which is *sp.* corresponds to a position of 226°·0 = 180° + (360 − 314°) = 226°.

61 Virginis. The R.A.S. MS. gives "Crystal at 160° 35′
180°
340° 35′"

It will be seen that the only possible position-angles where 180° can be properly *added* to the reading are 270° and 360°. From the table I have given we at once see that a circle reading of 160° 35′ must correspond to position-angles of either 19° 25′ or 199° 25′, and as the companion of 61 Virginis was obviously neither *sp* nor *sf*, the correct position angle of the Cycle is 19° 25′. Under no circumstances could a reading of 160° 35′ place the star in the *np* quadrant. Admiral Smyth has given the sum of 180° and the Micrometer reading, instead of the difference. This affords a satisfactory explanation of the discrepancy Mr. Burnham finds in the measures of this star. Further explanations of probably similar errors are offered in the subjoined notes to the following stars: χ Persei, γ Persei, 41 Sextantis, λ Herculis.

Mr. Burnham remarks upon the Cycle observations of the magnitudes and colours of small stars, which he says "are, to say the least, of very questionable accuracy in many instances." The following evidence with regard to colours is adduced in support of that statement.

52 Piscium. "The companion (14 m) is called 'deep blue.'"

76 M Persei. "The small star is described as 'dusky.'"

55 Andromedæ. "Smyth says the small star 'was only caught by intense attention' *and yet* calls it 'bluish.'"

53 Virginis. "Smyth called the small star (15 m) 'bluish.'"

72 Virginis. "Smyth does not seem to have been aware that this was one of the Struve stars and only refers to its being registered by ♅ but without measures. *And yet* the small star, which is only 11·5 of Σ's scale, was well enough seen to be described in the *Cycle* as 'violet tint.'"

163 P. XIII. "This faint star is called 'ash coloured' by Smyth."

α Boötis. "Very distant," called "lilac" by Smyth.

95 P. XIV. "This is described as 'a truly difficult object,' and best seen on averting the eye, *but called* 'bluish.'"

ε Ursæ Minoris. "The companion called 'pale blue.'"

η Cephei. "The companion is called 'dusky.'"

χ Delphini. "The small star is said to be 'pale lilac.'"

α Cephei. "The small star is described as 'pale sapphire.'"

312 P. XXI. "Smyth, who could see B only, calls it 'blue.'"

κ Andromedæ. "The small stars are called 'dusky' and 'ash coloured.'"

It is well known that Admiral Smyth devoted a great deal of attention to the colours of double stars; throughout the Bedford Catalogue no star is mentioned without some colour being given to it. In the "Speculum Hartwellianum" a whole chapter is devoted to the subject. This was subsequently reprinted with additions to form Smyth's tract on "Sidereal

Chromatics." In the preface to this tract (p. vi.) Smyth refers to the "Fancy Colours" he had given in the Bedford Catalogue, which he explains. His words are, "I will here cite a sample from my own practice, placing the *inexact epithets* which I have used in Roman print, and what was probably meant is expressed in Italics." Among the "inexact epithets" are found the above terms, "dusky,"* "ash-coloured," "lilac," and "sapphire," all of which are explained.

In his note to the star 55 *Andromedæ*, Mr. Burnham has omitted the important remark of Admiral Smyth with reference to the colour. He says ("Cycle," vol. ii. p. 43), "Is the intense blue which some of these mere points of light present an optical illusion?"

In "Cycle," vol. i. p. 302, and "Sidereal Chromatics," p. 15, Smyth says, "I have been much struck with the beautiful blue tint of several of the smallest stars visible in my telescope. This, however, may be attributed to some optical peculiarity." Again, in "Cycle," vol. ii. p. 290, he says, "B 15, intense blue; this last colour on so small an object is very striking, and an astronomical friend who examined it at my request with powerful means confirms both the tint and its intensity." In vol. ii. p. 115, he says:—"Σ. discovered the delicate companion C which had escaped the gaze of all other observers, and requires the most careful attention even to be perceived by occasional glimpses, but when seen, has a peculiar deep purple tint which strikes singularly on the eye from so excessively minute an object." In "Cycle," vol. ii. p. 522, he remarks that Sir John Herschel observes that a certain minute star "bears illumination well and is *therefore* blue." Mr. Proctor, in his "Orbs Around Us," p. 322, says, "The late Admiral Smyth, who thought he could recognise very decided blue tints among the minuter stars, expressed a doubt whether this might not be due to some idiosyncracy of his eyesight." And in their recently published "Manual of Astronomy," p. 452, Professor Newcomb and Professor Holden say that certain phenomena "are partly due to the physiological fact that *the fainter a star is the more blue it will appear to the eye.*" †

These quotations show that Admiral Smyth was perfectly well aware of the apparent anomaly in recording decided colours to minute points of light; and the important fact, stated by Professor Newcomb and Professor Holden in their "Astronomy," proves that such observations, as have been called in question by Mr. Burnham, were made by an eye very sensitive to colour, and are probably accurate.

In an interesting "Note on a Relation between the Colours and Magnitudes of the Components of Binary Stars" published in the *American Journal of Science*, June 1880, my friend Prof.

* "C is of the nondescript tint called 'dusky.'"—*Spec. Hart.* p. 265.

† "While the last gleamings of refracted light
Died in the fainting violet away."
Thomson, *Poem to the Memory of Sir Isaac Newton.*

Edward S. Holden says :—" We do not find *isolated* stars of decided green, blue, or purple colours. A few such have been recorded, but in most cases erroneously." To which a note is appended :—" For example, Admiral Smyth calls *α Lyræ* a *green* star." Admiral Smyth remarks in "Sidereal Chromatics," p. 17 : —" There is not an instance of a solitary green, purple, blue, or violet star being found." In the Bedford Catalogue, p. 423, and "Sidereal Chromatics," p. 33, the colour of *α Lyræ* is given "pale sapphire ; " and in the preface to "Sidereal Chromatics," p. vii., the word "pale" is stated to mean "deficient in hue," and the word "sapphire" to mean "blue tint." I have entirely failed in finding any reference to *α Lyræ* as a "green" star.

With regard to the stars under discussion a few conclusions may be safely drawn from the internal evidence afforded by Admiral Smyth's writings, independent of the proofs given by more recent micrometrical measurements. Firstly, nearly all these stars, and those with W. 1 attached, were never intended by Admiral Smyth to be considered as accurately micrometrically measured. The difficulties and misapprehensions that have arisen are entirely due to the improper manner in which such observations are printed with a decimal point and a cypher, and with a certain weight appended. It is plain on the face of it that observations of distances of one or two hundred seconds of arc in round numbers, without tenths or hundredths, cannot be the result of accurate micrometrical measurements. Secondly, the instrumental means of Admiral Smyth for these stars were not calculated to yield accurate results. The uncertainty and difficulty in using the spherical crystal micrometer is sufficiently demonstrated in the remarks I have made. The annular micrometer, used for estimating distances, consisted of a ring with a *radius* of $472''.5$, and it needs no further remark to show that estimations with such an instrument must be of the vaguest kind. And thirdly, we see by the quotations from the R. A. S. MSS. that Admiral Smyth was careless in his arithmetical reductions, and from the note to *β Aquarii* we perceive that his judgment in estimating distances was unsound.

The following notes on the stars speak for themselves in affording additional information and explanation of the results discussed by Mr. Burnham.

Additional Notes to Stars in Mr. Burnham's Class II.

β Cassiopeæ (p. 2).

Pos. W. 1. Dist. W. 1.

Smyth says :—" A bright star whose acolyte is so small that it is here rather *estimated* than measured." R. A. S. MS. gives distance "Δ A.R. $19^s.1 = 201''.0$."

147 H III. (p. 3).

Pos. W. 1. Dist. W. 1.

Apart from W. 1 being attached to each element, Smyth particularly mentions that this double star "is here *estimated*." The object is evidently inserted in the "Cycle" because it occupied the spot where Smyth sought for H's faint nebula, and it serves merely to introduce some remarks upon H's idea of nebulous fluid.

γ *Pegasi* (p. 5).

Pos. W. 1. Dist. W. 1.

The R. A. S. MS. gives the distance as "181″·0 or 182″·0."

ι *Ceti* (p. 6).

Pos. W. 1. Dist. W. 1.

Smyth expressly states:—"This is an excessively difficult object, being only discernible after long attention, and by occasionally averting the eye to another part of the field of view. The position and distance are therefore only the result of cautious *estimation*."

12 *Ceti* (p. 8).

AB. Pos. W. 1. Dist. W. 1.

AC. Pos. W. 2. Dist. W. 1.

Smyth calls it "a most difficult test object." "B is only discernible by the closest attention under favouring circumstances, though when once caught is tolerably well seen; the detail here given is therefore a *mere estimation*."

52 *Piscium* (p. 10).

Pos. W. 1. Dist. W. 1.

Smyth calls it "a most delicate object whose position and distance are carefully ["cautiously" in R. A. S. MS.] *estimated*."

δ *Andromedæ* (p. 11).

Pos. W. 2. Dist. W. 2.

α *Cassiopeæ* (p. 12).

Pos. W. 3. Dist. W. 3.

Smyth quotes H's measure and says:—"Which compared with mine does not show a greater difference of angle than might be expected from the proper motion of an object whose proximity is accidental, but the difference in distance is so remarkable, that it must be imputed to instrumental error." With regard to the

above remark Smyth adds in "Speculum Hartwellianum," p. 217: —"Meaning of course to impugn ♅'s 56″·17 of fifty years before, but it turned out instead to be a gross blunder of my own, only explicable on the supposition of counting a wrong revolution with the comb out of the field with the power used."

P. O. 146 (p. 14).
Pos. W. 3. Dist. W. 1.

H. 1046 (p. 17).
Pos. W. 1. Dist. W. 1.

The "Cycle" says:—"The double star here carefully *estimated*."

σ^2 *Piscium* (p. 24).
AB. Pos. W. 4. Dist. W. 2.
AC. Pos. W. 3. Dist. W. 1.

Mr. Burnham says:—"Smyth has *the extraordinary conclusion* that 'an inference of binarity is deducible from a comparison of the registered epochs of ♅ and himself,' whereas Smyth commences the sentence by saying '*No safe conclusion* can be deduced from the discrepancies observable in the position of A and B, as the object is most delicate.'"

R. A. S. MS. says:—"I requested Mr. Challis to make a diagram with the large telescope. His diagram confirms mine."

η *Ceti* (p. 27).
Pos. W. 1. Dist. W. 1.

β *Andromedæ* (p. 27).
Pos. W. 1. Dist. W. 1.

The star which Mr. Burnham calls D is clearly Smyth's companion. In the R. A. S. MS. the distance is given "Δ A.R. 8ˢ·8 W. 1, Pos. 299°·0 W. 1." The reduction of these elements gives 124″ or 125″ as the distance. The discrepancy is most probably a printer's error of 225 for 125.

H. 634 (p. 31).
Pos. W. 1. Dist. W. 1.

Smyth states that the position and distance are "*mere estimations*." They differ much from H.

42 ♅ VII. (p. 33).
Pos. W. 1. Dist. W. 1.

The R. A. S. MS. says:—"The position and distance of which are carefully *estimated*."

40 *Cassiopeæ* (p. 37).
Pos. W. 2. Dist. W. 1.

"A delicate object."

76 *M. Persei* (p. 40).
Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"The wide double star *estimated*."

107 *Piscium* (p. 41).
Pos. W. 2. Dist. W. 1.

Smyth says:—"The comes is so minute that light is inadmissible, the position is therefore gained by the spherical crystal micrometer, and the distance carefully *estimated*."

ζ *Ceti* (p. 43).
Pos. W. 3. Dist. W. 1.

R. A. S. MS. states:—"By annular micrometer nearly the same as wire. By spherical crystal Pos. 139° 24′ from 180°. Δ A.R. $7^{s}.8$."

At the bottom Pos. is given =40° 26′—a clerical error for 40° 36′. The distance is clearly computed from Δ A.R.

55 *Andromedæ* (p. 43).
Pos. W. 1. Dist. W. 1.

The small star was Smyth's minimum visibile; "its position and distance are therefore only *estimated*."

α *Trianguli* (p. 44).
Pos. W. 1. Dist. W. 1.

P. I. 222 (p. 47).
AD. Pos. W. 2. Dist. W. 2.

"This group is most difficult to observe, and the results are rather *estimations* than measures."

P. I. 227 (p. 48).
Pos. W. 5. Dist. W. 3.

χ *Persei* (p. 57).
AB. Pos. W. 2. Dist. W. 1.
AC. Pos. W. 5. Dist. W. 3.

Admiral Smyth's position-angle of AC is 43°·2, whereas Mr. Burnham makes it 136°·3 and remarks :—"It is not easy to see how Smyth could make such an error in his own observation, as there is no such mistake found anywhere else in the work." Smyth gives his observation of position W. 5, showing that he considered it had been made with care. A little consideration of the remarks upon the spherical crystal micrometer will show that 43°·2 is the *exact reading* of that instrument corresponding to a position-angle of 136°·8 thus : 180° − 43°·2 = 136°·8. It is most probable here that Smyth has given the reading of the circle without correction.

34 *M. Persei* (p. 66).
Pos. W. 2. Dist. W. 2.

The R. A. S. MS. says, "a double star *about* 15″ apart."

θ Persei (p. 67).
Pos. W. 1. Dist. W. 1.

γ^1 *Fornacis* (p. 72).
Pos. W. 2. Dist. W. 1.

Cycle says :—"As B vanishes under illumination its position is taken by the spherical rock-crystal micrometer, and the distance *estimated*."

γ Persei (p. 75).
Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—"May be made triple with C, but only measured H's, and that very slightly, for the evening was bad, and the object wide and unequal."

Smyth's position-angle is 226°·8. Mr. Burnham makes it 323°·9. It is probable that Smyth has given the reading of the spherical crystal micrometer without correction. The details of this instrument already given show that 226°·0 corresponds to a position-angle of 314°, which would be fairly satisfactory for a very slight measurement.

β Persei (p. 77).
Pos. W. 2. Dist. W. 1.

R. A. S. MS. has :—"The effulgence of the large star interferes with measures for distance."

52 *Arietis* (p. 77).
AD. Pos. W. 1. Dist. W. 1.

Smyth says :—"The whole [quadruple group] are of most

difficult measurement . . . the details of the latter two (C and D) being of course *mere estimations*."

τ^4 *Eridani* (p. 79).

Pos. W. 1. Dist. W. 1.

α *Persei* (p. 80).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"A is so brilliant as to render measures difficult, they are *mere estimations*."

δ *Persei* (p. 82).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. says:—"By rock crystal Pos. 224° 40′ − 90° =315°·0" (this should be +90°). "Dist. Δ A.R. $7^s{\cdot}0$."

The reduction of the Δ A.R. differs only 3″ from Mr. Burnham's measure; a probable error by Smyth in reducing his observation.

19 *Pleiadum* (p. 84).

Pos. W. 2. Dis. W. 1.

P. IV. 257 (p. 108).

AC. Pos. W. 2. Dist. W. 2.

R. A. S. MS. says:—"Δ A.R. AC=$4^s{\cdot}3$."

66 *Eridani* (p. 111).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. has for distance "Δ A.R.=$0^s{\cdot}4$."

β *Eridani* (p. 111).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. has for distance "Δ A.R.=$4^s{\cdot}2$."

H. 3269 (p. 112).

Pos. W. 2. Dist. W. 2.

H. 3272 (p. 118).

Pos. W. 1. Dist. W. 1.

γ *Orionis* (p. 121).

Pos. W. 3. Dist. W. 2.

R. A. S. MS. has for distance "Δ A.R. $3^s{\cdot}6$."

109 *P. Orionis* (p. 124).
Pos. W. 1. Dist. W. 1.

β Leporis (p. 125).
Pos. W. 1. Dist. W. 1.

R. A. S. MS. has for distance "Δ A.R. $13^{s}{\cdot}1$."
Mr. Burnham says:—"There are two 10 mag. stars in this quadrant." The R. A. S. MS. diagram shows both these stars.

ε Orionis (p. 134).
Pos. W. 1. Dist. W. 1.

R. A. S. MS. has for distance "Δ A.R. $9^{s}{\cdot}8$."

124 *Tauri* (p. 135).
BC. Pos. W. 7. Dist. W. 3.
AB. „ W. 1. „ W. 1.
AD. „ W. 1. „ W. 1.

In the R. A. S. MS. it is evident that BC was measured, but AB and AD were only arrived at by Δ A.R. The MS. gives "Δ A.R. A and B$=5^{s}{\cdot}5$, Δ A.R. A and D$=1^{s}{\cdot}0$."

ν Aurigæ (p. 139).
Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"*Mere estimations.*"

78 *M. Orionis* (p. 138).
Pos. W. 2. Dist. W. 1.

37 *M. Aurigae* (p. 140).
Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"It is preceded by the double star here measured *or rather estimated.*"

β Aurigae (p. 143).
Pos. W. 2. Dist. W. 2.

With regard to the measure by Challis given by Mr. Burnham, the R. A. S. MS. has:—"I requested Mr. Challis to look to it (print his results)."

θ Aurigae (p. 143).
Pos. W. 2. Dist. W. 1.

35 *Camelopardi* (p. 144).
Pos. W. 2. Dist. W. 1.

5 *Lyncis* (p. 148).

AB. Pos. W. 1. Dist. W. 1.
AC. „ W. 7. „ W. 2.

R. A. S. MS. says:—"A and C are the stars which constitute H VI. B seems not to have been seen till Σ announced it. It is seen readily in the spherical crystal micrometer, illumination being impracticable."

μ *Geminorum* (p. 148).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R. $5^{s}\cdot4$."

ζ *Canis Majoris* (p. 149).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R. $6^{s}\cdot7$," and says:—"Piazzi's 80 and 81 Hora VI. by whose observations reduced they would be for 1800 Pos. 337°·0. Dist. 178·0."

It is worthy of notice that though Piazzi's reduced observations agree with Mr. Burnham's results, and were evidently very good, Admiral Smyth does not appear to have made his agree with them.

β *Canis Majoris* (p. 150).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ. A.R. $4^{s}\cdot7$."

10 *Monocerotis* (p. 151).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. diagram shows that Mr. Burnham's B is the star measured, as C is also given.

γ *Geminorum* (p. 154).

AB. Pos. W. 1. Dist. W. 1.
AC. „ W. 1. „ W. 1.

The Cycle gives the distance as $75^{s}\cdot0$ and $110^{s}\cdot0$, as if seconds of time, and says:—"Followed nearly on the parallel Δ A.R. 40^{s} by a neat 9 mag. star." We may fairly presume that the distances AB and AC were determined by Δ A.R.

α *Canis Majoris* (p. 158).

Pos. W. 1. Dist. W. 1.

H. 740 (p. 165).

Pos. W. 2. Dist. W. 1.

ζ *Geminorum* (p. 169).

AB. Pos. W. 4. Dist. W. 2.
AC. „ W. 1. „ W. 1.

R. A. S. MS. says "Distance Δ A.R. A and B=1″·8. Δ A.R. A and C=5″·0."

δ *Canis Majoris* (p. 170).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says "Angle by spherical crystal=314°·0−90°. Distance=Δ A.R.=13″·5."

The reading of 314°·0 on the spherical crystal micrometer corresponds to a position of 226°, not 224° as given by Smyth.

30 *Canis Majoris* (p. 174).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says:—"Distance = Δ A.R. 5″·6=85″·0. A bright white star in a rich cluster of small ones, and with its comes very splendid among the star dust." The two near companions mentioned by Mr. Burnham are probably included in the "star dust."

η *Canis Majoris* (p. 175).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R.=13″·4."

β *Canis Minoris* (p. 176).

AB. Pos. W. 2. Dist. W. 1.
AC. „ W. 2. „ W. 1.

R. A. S. MS. says:—"See Baron de Zach in vol. xii."

H. 2435 (p. 195).

Pos. W. 4. Dist. W. 1.

P. VIII. 81 (p. 197).

Pos. W. 2. Dist. W. 1.

It is to be remarked that Smyth does not agree with the earlier measure of Struve.

θ *Cancri* (p. 198).

Pos. W. 3. Dist. W. 1.

ψ *Leonis* (p. 220).

Pos. W. 2. Dist. W. 1.

41 *Sextantis* (p. 234).

AB. Pos. W. 1. Dist.

AC. „ W. 2. „ W. 1.

Smyth says:—"The *estimated* angle and distance (AB) are next to *mere guesses*." R. A. S. MS. says:—"Mere estimations." A and C. Pos. 120°·4 W. 2. Mr. Burnham gives it 72°·5. The remarks on the spherical crystal micrometer show that we must be prepared for numerous mistakes in readings and their reduction. If the above position-angle 120°·4 be really the reading of that micrometer it represents a true position-angle of 59°·6, which is close enough for a mere guess.

β *Ursæ Majoris* (p. 237).

Pos. 172°·6 W. 2. Dist. 75·0 W. 1.

R. A. S. MS. says the star is *up*, but the position makes it *nf*. The diagram in the MS. shows the companion about 272°, and at a much greater distance than 75″·0. I am, however, inclined to think that the diagrams in the MS. have been made from the recorded observations, and are not eye-sketches.

δ *Leonis* (p. 244).

Pos. W. 1. Dist. Δ A.R. 4ˢ·9 W. 1.

δ *Crateris* (p. 248).

Pos. W. 1. Dist. Δ A.R. 19ˢ·8 W. 1.

δ *Ursæ Majoris* (p. 262).

Pos. W. 2. Dist. Δ A.R. 20ˢ·4 W. 1.

8 *Can. Ven.* (p. 272).

Pos. W. 1. Dist. W. 1.

β *Corvi* (p. 271).

Pos. W. 1. Dist. W. 1.

Smyth says:—"Position and distance here *estimated*."

δ *Virginis* (p. 286).

Pos. W. 2. Dist. Δ A.R. 5ˢ·8 W. 2.

R. A. S. MS. has "Δ A.R. W. 1."

ε *Virginis* (p. 290).

Pos. W. 4. Dist. W. 2.

R. A. S. MS. says:—"A star with a very minute distant comes."

53 *Virginis* (p. 292).

Pos. W. 1. Dist. W. 1.

Smyth says:—"As I could only catch a sight of B by gleams with the equatorial clock driving the telescope, the above results are but *estimations*."

61 *Virginis* (p. 294).

♅	Pos. 345°	Dist. 73″·25.
Smyth	„ 340°·6 W. 2.	Δ A.R.. 2ˢ·8 W. 1.
β	„ 24°·8	189″·26.

R. A. S. MS. has in pencil in Smyth's writing:—Chrys. 160° 35′ W. 2;" then below—

" = 160° 35′
180
340 35"

and at bottom "=340°·6."

This discrepancy has been explained in remarks on the spherical crystal micrometer. Mr. Burnham, who has perhaps viewed the above figures under the aspect of a preconceived theory, says:—"There is a large error in the *repetition of* ♅'s *angle* by Smyth." The correct reduction here is 180° − 160° 35′ = 19° 25′.

α *Virginis* (p. 296).

Pos. W. 2. Dist. Δ A.R. = 19ˢ·4 W. 2.

μ *Hydræ* (p. 300).

Pos. W. 2. Dist. W. 2.

72 *Virginis* (p. 301).

Pos. W. 3. Dist. W. 1.

R. A. S. MS. says:—"Measures of distance out of the question with my means; it is therefore a *mere estimation*."

75 *Virginis* (p. 303).

Pos. W. 1. Dist. W. 1.

Cycle says:—"This object was merely looked at from being among ♅'s sweeps, otherwise it is too difficult to measure and *too wide for tolerable estimation*."

163 *P.* XIII. *Can. Ven.* (p. 306).

Pos. W. 2. Dist. W. 2.

Cycle says :—"This was selected for a trial of the spherical crystal micrometer on the angle of position by getting the double image of A in a line towards B. The distance, being too great for the value of the scale, was obtained by Δ A.R."

R. A. S. MS. says nothing about Δ A.R. but gives for distance "68°·0 W. 2. 70·0 est."

171 *P.* XIII. *Virginis* (p. 308).

Pos. W. 4. Dist. W. 2.

R. A. S. MS. says :—"Though the companion is so minute, it bears the red illumination pretty fairly. Seen well with the rock-crystal prism."

85 *Virginis* (p. 309).

Pos. W. 1. Dist. W. 1.

Cycle says :—"This most difficult object was merely examined from being one of Herschel's sweeps. It is the *minimum visibile* of my telescope, and therefore impossible to measure."

η Bootis (p. 311).

Pos. W. 4. Dist. W. 2.

τ Virginis (p. 312).

Pos. W. 5. Dist. W. 3.

α Bootis (p. 315).

Pos. W. 2. Dist. W. 1.

Cycle gives for distance Δ A.R. $15^{s}\cdot1$. W. 1.
The R. A. S. MS. appends "$=298''\cdot0$."

P. XIV. 95 (p. 322).

Pos. W. 2. Dist. W. 1.

Smyth says :—"The position was approximated by the spherical rock-crystal micrometer, but the distance being too wide for its range it was *merely estimated*." R. A. S. MS. gives distance "$=40''\pm$ est."

5 *Ursæ Minoris* (p. 323).

Pos. W. 2. Dist. W. 1.

β Ursæ Minoris (p. 330).

Pos. W. 1. Dist. W. 1.

δ Bootis (p. 338).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R. $7^{s}\cdot9$."

ι *Draconis* (p. 344).
Pos. W. 2. Dist. W. 1.

α *Serpentis* (p. 348).
Pos. W. 1. Dist. W. 1.

R. A. S. MS. has, "excessively difficult; *merely estimated.*"

ζ *Ursæ Minoris* (p. 351).
Pos. W. 2. Dist. W. 1.

δ *Ophiuchi* (p. 356).
Pos. W. 1. Dist. W. 1.

υ *Coronæ* (p. 358).

AB.	Pos. W. 6.	Dist. W. 4.
AC.	„ W. 8.	„ W. 5.
BD.	„ W. 2.	„ W. 1.

R. A. S. MS. says:—"Bears illumination badly."

ν^2 *Coronæ* (p. 362).
Pos. W. 1. Dist. W. 1.

η *Draconis* (p. 365).
Pos. W. 1. Dist. W. 1.

β *Herculis* (p. 366).
Pos. W. 2. Dist. W. 1.

η *Herculis* (p. 372).

AC.	Pos. W. 2.	Dist. W. 1.
AB.	„ W. 1.	„ W. 1.

Cycle says:—"Tolerable estimations of an egg-shaped object.'
R. A. S. MS. gives AC distance ="Δ A.R. $9^s.5$."

Mr. Burnham says:—"At one time Σ suspected the large star to be a close pair, but afterwards rejected it as single. Of the correctness of this conclusion there has never been any doubt. Smyth measured the angle and distance in 1842." Mr. Burnham has ignored the explanation given by Smyth in *Speculum Hartwellianum* on this subject. He says (p. 275), with regard to AB.:—"This first-class specimen of Σ's Vicinissimæ was gazed at in 1842 till an impression arose that it was slightly elongated, but un-notchable, and therefore under half a second in distance, *but the weights assigned show that the idea was vague, and the conclusions worth next to nothing.*"

η *Ophiuchi* (p. 379).
Pos. W. 1. Dist. W. 1.

35 *Ophiuchi* (p. 381).

AC. Pos. W. 4. Dist. W. 2.

ϵ *Ursæ Minoris* (p. 381).

Pos. W. 1. Dist. W. 1.

λ *Herculis* (p. 390).

Cycle Pos. = 295°·0 W. 1. Dist. Δ A.R. 23ˢ·4 W. 1.

β ,, = 251°·8

Smyth's position makes B *np*, but in R. A. S. MS. it is given *sp*, where Mr. Burnham proves the star to be. We may surmise that this is another case where the reading of the spherical crystal micrometer has been inserted without reduction. 295°·0 would correspond to a position-angle of 245°·0.

γ *Draconis* (p. 400).

Pos. W. 1. Dist. Δ A.R. 9ˢ·7 W. 1.

τ *Ophiuchi* (p. 402).

Pos. AC. { W. 2. / W. 3. Dist. { W. 1. / W. 2.

Smyth says he made a hopeless scrutiny of AB, and "making nothing of it, I noted the star C *as a future reference*."

18 M. *Clyp. Sobieski* (p. 415).

Pos. W. 1. Dist. W. 1.

Smyth has probably given the position-angle BA instead of AB.

H. 2002 (p. 414).

Pos. W. 1. Dist. W. 1.

χ *Draconis* (p. 421).

Pos. W. 1. Dist. Δ A.R. 14ˢ·6 W. 1.

2 *Aquilæ* (p. 426).

Pos. W. 2. Dist. W. 1.

Smyth says:—"It is a difficult object to manage, from the brightness of A."

5 *Aquilæ* (p. 427).

AC. Pos. W. 1. Dist. W. 1.

R. A. S. MS. has:—"C is very minute, so that its place is only *estimated* for placing it."

25 M. *Sagittarii* (p. 420).

Pos. W. 1. Dist. W. 1.

Mr. Burnham's difficulty in identifying the pair is possibly explained in R. A. S. MS. which says :—"A knot of from 10th to 13th mag. stars are *between* two stars of 8th. This is the position fixed." Note in MS. says :—"See Blue Book, p. 55, for description and position, &c." The diagram in MS. does not accord with the Cycle position-angle.

110 *Herculis* (p. 428).

Pos. W. 1. Dist. W. 1.

Smyth says :—"The acolyte was caught by gleams on steady gazing in a darkened field; it is certainly a *minimum visibile* in my telescope. The details are of course but *mere estimations*."

P. XVIII. 197 (p. 430).

Pos. W. 3. Dist. W. 1.

ν^1 *Lyræ* (p. 432).

AB.	Pos. W. 2.	Dist. W. 1.
AC.	„ W. 8.	„ W. 8.
CD.	„ W. 1.	„ W. 1.

R. A. S. MS. says :—"Δ A.R. AC$=3^{s}{\cdot}2$." This with W. 8 is rather inexplicable.

Cycle gives :—"As B and D are too small to bear any illumination the places above given are mere, though careful, *estimations*."

β *Lyræ* (p. 433).

AB.	Pos. W. 9.	Dist. W. 9.
AC.	„ W. 2.	„ W. 1.
AD.	„ W. 1.	„ W. 1.

Mr. Burnham states that this star is inserted "as an illustration of the *apparent accuracy* of the measures of Smyth when the star had been previously observed. The other companions, C and D, had not been measured at this time by other observers." As a matter of fact, the measures of C and D have W. 1, and were therefore not intended to be considered as accurate measures.

2024 H. *Aquilæ* (p. 437).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—"During the measures a bright Aurora Borealis."

γ *Lyræ* (p. 439).

Pos. W. 1. Dist. Δ A.R. W. 1.

P. XVIII. 299 (p. 440).

Pos. W. 1. Dist. W. 1.

Smyth says :—"The acolyte is of course too minute to admit of illumination, and its place is therefore *merely estimated*."

ζ *Aquilæ* (p. 441).

Pos. W. 1. Dist. Δ A.R. W. 1.

Smyth says :—"As the comes defies illumination, the position is inferred by pointing the double image of A in the rock-crystal micrometer to the direction of B."

δ *Draconis* (p. 445).

Pos. W. 1. Dist. Δ A.R. W. 1.

δ *Aquilæ* (p. 446).

Pos. W. 1. Dist. W. 1.

Smyth says :—"The position and distance were obtained by *estimations* made in a dark field for the angle, and the Δ A.R. by a bar for the distance."

R. A. S. MS. gives Dist. Δ A.R.$=6^{s}\cdot6=96''\cdot50$

56 *Aquilæ* (p. 462).

Pos. W. 4. Dist. W. 1.

Smyth says :—"The angle of position was readily obtained (W. 4), but the distance is a mere estimation (W. 1) by Δ A.R. over a small bar in the eyepiece expressly fitted for such cases."

β *Aquilæ* (p. 464).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives Dist. ="Δ A·R.$=3^{s}\cdot7=175''\cdot0$."

γ *Aquilæ* (p. 455).

Pos. W. 1. Dist. W. 1.

α^{1} *Capricorni* (p. 472).

B*b*. Pos. W. 3. Dist. W. 1.

A*a*. ,, W. 2. ,, W. 1.

There is a strange and unexplained difference in the accuracy of the distances B*b* and A*a*, both having the same W. 1.

M. 29 *Cygni* (p. 478).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—"Dist. Δ A.R. of A and B about $4^{s}\cdot9$."

1 *Aquarii* (p. 483).

AB. Pos. W. 1. Dist. W. 1 (in R. A. S. MS.).

AC. „ W. 1. „ no weight given.

α *Delphini* (p. 483).

Pos. W. 1. Dist. W. 1.

α *Cygni* (p. 485).

Pos. W. 1. Dist. W. 1.

η *Cephei* (p. 488).

Pos. W. 1. Dist. W. 1.

χ *Delphini* (p. 490).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. has :—"The position and distance of A and B are by comparative *estimations*." Position is given "20°59′ (by crystal)."

59 *Cygni* (p. 492).

Pos. W. 2. Dist. W. 1.

R. A. S. MS. has :—"The colours were proved for me by Mr. Challis."

ζ *Cygni* (p. 497).

Pos. W. 1. Dist. W. 1.

Smyth probably measured distance with micrometer, for R. A. S. MS. gives distance="104″·79=150″·0."

α *Cephei* (p. 499).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives distance "Δ A.R.=9ˢ·8 W. 1; 8ˢ·8 W. 2." The small star is called "pale blue," not "pale sapphire."

β *Aquarii* (p. 502).

Pos. W. 2. Dist. W. 1.

Cycle says :—"This is a most difficult object, and one requiring the utmost delicacy of treatment to procure even an estimation of."

R. A. S. MS. says :—"The angle of position by the spherical crystal micrometer, and the distance an *estimation or comparison*," adding "H's is certainly too wide."

We have here an instance to prove that Smyth's judgment was faulty; Mr. Burnham has proved that H's measure was very slightly in error.

ϵ *Pegasi* (p. 507).

AB. Pos. W. 1. Dist. W. 1.
AC. „ W. 8. „ W. 8.

P. XXI. 312 (p. 509).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—" Measurement was out of the question, the angle and distance are therefore by careful *estimation*."

20 *Pegasi* (p. 509).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—" Position and distance are *mere estimations*."

α *Aquarii* (p. 510).

Pos. W. 1. Dist. W. 1.

π^1 *Pegasi* (p. 512).

AB. Pos. W. 3. Dist. W. 2.
AC. „ W. 3. „ W. 1.

2 *Lacertæ* (p. 515).

Pos. W. 1. Dist. W. 1.

Smyth says :—" The *estimations* were carefully made."

ζ *Pegasi* (p. 520).

Post. W. 1. Dist. W. 1.

It is not easy to reconcile Smyth's position 15°·5 with Mr. Burnham's 137°·8. 15°·5 in the spherical crystal micrometer corresponds to an angle of 164°·5.

ξ *Pegasi* (p. 522).

AB. Pos. W. 1. Dist. W. 1.
AC. „ W. 2. „ W. 1.

Cycle says :—" B only to be caught under intense gazing with the most favourable circumstances of atmosphere and instrument. I added C for identification."

β *Pegasi* (p. 527).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. says :—" B is readily seen at intervals."

4 *Cassiopeæ* (p. 534).

AB.	Pos. W. 6.	Dist. W. 2.
AC.	,, W. 2.	,, W. 1.
CD.	,, W. 1.	,, W. 1.

R. A. S. MS. gives distance "Δ A.R. A and C=$25^{s}\cdot3$, A and B= $7^{s}\cdot4$. The companion of C is very faint at times."

The great discrepancy in the position-angle CD is not explained.

ι *Piscium* (p. 536).

Pos. W. 1. Dist. W. 1.

R. A. S. MS. gives:—"Position $140^{\circ}\cdot30=39^{\circ}\cdot30$; Δ A.R. $10^{s}=199''$."

This position is $180^{\circ}-140^{\circ}\ 30'$.

See remarks on spherical crystal micrometer.

κ *Andromedæ* (p. 536).

AB.	Pos. W. 1.	Dist. W. 1.
AC.	,, W. 2.	,, W. 1.

Cycle says:—"The *estimations* here given may at best be only ranked as *comparative guesses.*"

R. A. S. MS. says:—"Principally guesses."

P. XXIII. 171 (p. 537).

AB.	Pos. W. 3.	Dist. W. 1.
AC.	,, W. 1.	,, W. 1.

R. A. S. MS. gives "Distance Δ A.R. $8^{s}\cdot4$ A and C."

Bocking, Braintree,
1880, *June.*

On the Variability of B.A.C. 2472. By W. S. Franks, Esq.

In the May Number of the *Monthly Notices*, Mr. Tebbutt calls attention to the magnitude of this star, which is P vii. 114. On referring to my note-book, I find that it was observed on January 28, 1878, and there estimated as of the seventh magnitude. It is marked eighth magnitude in the large maps of the S.D.U.K.; but it is not contained in the *Uranometria Nova* of Argelander.

Leicester,
1880, *June* 10.

Elements of Comet Schäberle (1880, *b*). By T. H. Safford, Esq.

T. 1880, July, 1·2602 M. T. Washington.

ω	144° 59′·70
Ω	257 11·10
i	123 4·73
log q	0·25894

They were computed from observations made at

Ann Arbor	April 9
Vienna	April 13
Ann Arbor	April 27
Ann Arbor	May 4

In their computation all the small corrections (aberration, parallax) were neglected: but the ratio of curtate distances at the first and last observations was so varied that the intermediate places were represented as closely as possible, while the extreme places were perfectly represented.

The following are the values of computed — observed, latitudes and longitudes.

	$\Delta\lambda \cos\beta$	$\Delta\beta$
April 13	+0′·07	−0′·16
27	−0·17	+0·11

The present calculation was peculiar in this, that the two values of log M obtained by Olbers' approximation were naturally quite different, and gave at once limits for the employment of the *regula falsi*. I think it is often well in calculating the orbit of a comet to employ four observations in order to avoid the danger of mistakes.

Observatory, Williams College,
Williamstown, Mass.,
1880, June 3.

Elements and Ephemeris of Schäberle's Comet (1880, *b*). By M. G. Bigourdan.

(*Extracted from the 'Comptes Rendus,' Tome xci., Nos. 2 and 3.*)

The following elements are derived from three normal places formed respectively from observations on

(1) April 8 (Paris); 10 (Pola); 11 (Strasburg); 13 (Vienna).
(2) April 26 (Paris); 28 (Rome); 29 and 30 (Paris).
(3) May 14, 16, 17, 18 (Paris).

T 1880, July 1·83846, Paris M.T.

ϖ 42° 30′ 56″·1
Ω 257 15 13·3 } Mean Equinox 1880·0.
i 123 3 36·1

log q 0·258474.

Ephemeris for Paris Midnight.

1880	R.A. h m s	Dec. ° ′ ″	Log. Δ	Brightness
Aug. 15·5	6 59 59·8	29 1 16 N	0·406448	0·75
17·5	7 0 4·8	28 6 43	0·402808	0·75
19·5	7 0 5·9	27 51 45	0·398981	0·76
21·5	7 0 2·9	27 16 20	0·394969	0·77
23·5	6 59 55·4	26 40 24	0·390774	0·78
25·5	6 59 43·3	26 3 56	0·386398	0·78
27·5	6 59 26·4	25 26 52	0·381839	0·79
29·5	6 59 4·4	24 49 9	0·377100	0·80
31·5	6 58 37·0	24 10 45	0·372184	0·81
Sept. 2·5	6 58 3·9	23 31 35	0·367093	0·83
4·5	6 57 24·9	22 51 36	0·361833	0·84
6·5	6 56 39·5	22 10 44	0·356409	0·85
8·5	6 55 47·5	21 28 57	0·350825	0·86
10·5	6 54 48·6	20 46 9	0·345088	0·88
12·5	6 53 42·3	20 2 16	0·339207	0·89
14·5	6 52 28·4	19 17 16	0·333192	0·90
16·5	6 51 6·5	18 31 4	0·327051	0·92
18·5	6 49 36·3	17 43 34	0·320793	0·94
20·5	6 47 57·3	16 54 44	0·314434	0·95
22·5	6 46 9·1	16 4 29	0·307987	0·97
24·5	6 44 11·3	15 12 45	0·301465	0·99
26·5	6 42 3·4	14 19 27	0·294889	1·01
28·5	6 39 45·0	13 24 32	0·288278	1·03
30·5	6 37 15·7	12 27 57	0·281658	1·04
Oct. 2·5	6 34 35·0	11 29 39	0·275054	1·06
4·5	6 31 42·4	10 29 35	0·268493	1·08
6·5	6 28 37·6	9 27 43	0·262007	1·10
8·5	6 25 20·1	8 24 3	0·255635	1·12
10·5	6 21 49·6	7 18 36	0·249414	1·14
12·5	6 18 5·9	6 11 21	0·243386	1·16
14·5	6 14 8·8	5 2 24	0·237588	1·17

The brightness on April 6·5 (near the time of discovery) is taken as unity.

On the Star No. 894 Greenwich First Seven Year Catalogue.

By M. A. Wagner.

(*Communicated by the Astronomer Royal.*)

In his paper on the supposed transneptunian planets Professor Forbes refers to the star No. 894 of the Greenwich Seven Year Catalogue which he thinks might have been one of the hypothetical planets. The star has been observed twice in 1857, April 15 and 16. The results of the individual observations are:—

	App. A.R.	App. N.P.D.	Mean A.R. and N.P.D. 1857·0.
	h m s	° ′ ″	s ″
Apr. 15	11 19 6·74	85 2 35·41	4·95 26·16
16	6·99	32·86	5·21 23·64

The difference between the two observations is not greater than is consistent with the note "very faint," added to the second observation. If, indeed, these two observations have belonged to a planet moving in an orbit with a mean distance of 100, and a time of revolution of 1,000 years the Right Ascension of the object ought to have diminished by about $1^s.6$, and the N.P.D. to have increased by a number of seconds. Hence it appears that these observations cannot belong to a planet.

It might be inferred from Professor Forbes' remark that the alleged star is now wanting in the heavens, and, indeed, there is no star to be found at this place in the *Bonner Durchmusterung*. But the *Durchmusterung* is complete only to the ninth magnitude, and a star designated as "very faint" in the Greenwich Transit Circle probably will be of a less magnitude.

Our 6-inch refractor, however, and the heliometer show the star in its proper place. It is perhaps a little less than tenth magnitude. By comparing it with the two stars W. XI. 233 = Schj. 4117 and W. XI. 258 = Schj. $412\frac{6}{7}$, I deduced from observations of May 5 its mean position 1880·0—

$$\alpha = 11^h\ 20^m\ 16^s.2 \qquad \delta = +4^\circ\ 49'\ 58''$$

The Greenwich position from the Seven Year Catalogue reduced to 1880·0, gives—

$$\alpha = 11^h\ 20^m\ 16^s.20 \qquad \delta = +4^\circ\ 50'\ 1''.4$$

In the *Atlas Ecliptique de l'Observatoire de Paris* this star also seems to exist, although a little misplaced. The map for 1852·5 has a star of the eleventh magnitude in $11^h\ 18^m\ 56^s + 5^\circ\ 3'$, which reduced to 1880·0 gives $11^h\ 20^m\ 21^s + 4^\circ\ 54'$, but in this declination there is no star; the general configuration, how-

ever, agrees very well with the heaven. Therefore, if the original entries of the Greenwich Observatory do not state expressly that the star was brighter than tenth magnitude on April 15, 1857, there seems to be no reason for supposing it a variable star.

Pulkowa, 1880, *May* 12.

Coincidence of Sun-spots and Auroræ in Olden Time.
By the Rev. S. J. Johnson.

As with a slowly increasing number of Sun-spots at the present time we may assume the return of fine auroral displays to be drawing near, it may not be devoid of interest to examine what coincidence may be traced between the auroræ seen in this land by our forefathers and the few recorded instances we have of Sun-spots in early times. Meagre though our data may be, it is by no means difficult to trace this coincidence. The ease with which this may be done is more remarkable from the fact that, according to Wolf's tables, there are instances laid down of sixteen years between the assumed maxima of solar spots, and at other times less than half this period.

In a communication to *Nature*, January 13, 1870, an endeavour was made to show that the epochs at which Sun-spots were seen before the invention of the telescope were coincident with the maximum periods of the Sun-spot cycle. Thus it was remarked—

From	321	A.D. to	1860	we have	139	periods of	11·072 +	years each
„	321	„	807	„	44	„	11·045	„
„	807·22	„	840·5	„	3	„	11·093	„
„	840·5	„	1096	„	23	„	11·109	„
„	1096	„	1161	„	6	„	10·833	„

But a great number of spots are on record, chiefly in China, as seen with the naked eye, in addition to those mentioned in the said article.

The first instance in our own country occurs less than twenty years after the first recorded eclipse, being the mention by Matthew of Westminster of "fiery lances in the air," in the year 555. The Chinese do not record spots close to this date, but in 577, which would give two periods of eleven years each.

The following instances will be found in the *Chronicon Scotorum* and the *Anglo-Saxon Chronicle*.

A.D. 660. "In the summer, the sky was seen to burn." *Chronicon Scotorum*.

Employing the data above referred to, it will be seen that from 660

to 807 there are thirteen periods of about $11\frac{1}{3}$ years each. The following evidently occurred about the time of the next Sun-spot maximum.

670. "A thin and tremulous cloud in the form of a rainbow appeared at the fourth watch of the night of the fifth day before Easter Sunday (stretching) from east to west in a clear sky. The moon was turned into blood." *Chron. Scot.* The expression "clear sky" may be understood comparatively as it is now generally assumed that the aurora is hardly ever visible in a perfectly clear sky, and often appears mingling with cirrous haze. On one occasion only, I observed an aurora when not a trace of cloud was perceptible. In the instance of A.D. 670, the auroral arch appears to have been very marked, as we witnessed in 1869, 1870, and 1872. The expression concerning the Moon in other parts of the early chronicles, always indicates a total eclipse. On this occasion, however, there was no eclipse, but the Moon must have appeared suffused with the auroral vapour. It is worthy of note that on March 19, 1848, when the Moon shone with such vivid redness during the total phase, though deeply immersed in the shadow of the earth, instead of the ordinary dull copper tint, an aurora was prevailing.

710. "A bright night in autumn," *Chron. Scot.* This somewhat obscure expression seems best explained by an aurora; and if so, would happen four ten-year periods after the luminous arc of 670.

773. "In this year a red cross appeared in the heavens after sunset." *Angl.-Sax. Chron.* It is probable an auroral light is referred to here. The interval, from 773 to 807, when spots were particularly noted, gives three periods of $11\frac{1}{3}$ years each.

793. "In this year, dire forewarnings came over the land of the Northumbrians, and miserably terrified the people; there were excessive whirlwinds and lightnings, and fiery dragons were seen flying in the air." *Sax. Chron.* The "fiery dragons" may give the idea of red auroral streamers. But this would not synchronise with an assumed Sun-spot maximum, and may be best explained by a meteoric display.

890. "The heavens appeared to be on fire at night on the Kalends of January." *Chron. Scot.* There would be five ten-year periods from 840, when Sun-spots were often noticed during the summer months. As the intervals are periods of ten years, not of half or two-thirds of that number, we may regard the coincidence as quite satisfactory, though eleven years is generally assumed as the average Sun-spot interval. Again, conspicuous auroræ are noticed a year or two at least on each side of the maximum time.

944. "Two fiery columns were seen a week before Allhallowtide which illuminated the whole world." *Chron. Scot.* We have here ten average periods of 10·4 years from 840.

979. "A bloody cloud in likeness of fire was seen oftentimes; and that was most apparent at midnight, and was coloured in various rays. Then, when it was about to dawn, it glided away." *Sax. Chron.* We may suppose either twelve average periods of 11·6 years, or thirteen of 10·7 years after 840.

1098. "Before S. Michael's Mass the heaven appeared almost all night as if it were burning." *Sax. Chron.* Two years before this, solar spots were seen with the naked eye. It would, therefore, happen not long after a Sun-spot maximum.

1117. "In the night of the XVIIth of the Kal. of January (Dec. 16th), the heaven was seen very red, as if it were a conflagration." *Sax. Chron.* The interval from 1096 gives two periods of 10·5 years, and in the following year, 1118, the Chinese record Sun-spots.

1122. "There were many shipmen on the sea, and on (fresh) water, who said that they saw in the north-east a great and broad fire near the earth, which at once waxed, in length, up to the sky, and the sky separated into four parts, and fought against it, as if it would quench it; but the fire nevertheless waxed up to the heavens. The fire they saw in the dawn, and it lasted so long till it was light over all. That was on the day the VIIth of the Ides of December (Dec. 7th)." *Sax. Chron.* The account reminds one of Stowe's description of a fine aurora in the time of Queen Elizabeth. It was but a short interval from the last display in 1117, but the Chinese records speak of Sun-spots being seen the next year, in 1123. We must therefore conclude that the Sun's disc was in a very active state about this time, and according to Dr. Wolf there have been intervals of only $7\frac{1}{2}$ years in modern times between the maxima.

1131. "This year after Christmas, on a Monday night, at the first sleep, the heaven was, on the north side, all as though it were a burning fire, so that all who saw it, were so affrighted as they never were before. That was on the IIIrd of the Ides of January (Jan. 11th)." *Sax. Chron.* An interval of nine years from the last. It will be observed that it was witnessed three ten-year periods before the spot of the Spanish Moor Averroës (1161). The Chinese have numerous records of solar spots in this century, the 12th, and also in the 4th. The *Chronicon Scotorum* and *Anglo-Saxon Chronicle* contain no mention of these phænomena.

Abbenhall Rectory, Gloucester,
1880, *June* 9.

On the Refractive and Dispersive Powers of various Samples of Glass. By Mr. J. G. Lohse.

(*Communicated by Lord Lindsay.*)

With a view to obtaining a knowledge of the refractive and dispersive powers of various sorts of glass now in actual use, the prisms have recently been examined at Dun Echt Observatory by Mr. J. G. Lohse. Prisms 1 to 8 were kindly lent by Mr. Hilger with the option of purchasing any or all of them except No. 8, which was a beautiful sample prism of an object-glass in course of construction. Nos. 9 and 10 were supplied by Mr. Grubb as closely resembling the lenses of the Dun Echt 15·06-in. refractor.

The indices of refraction are for vacuum-glass.

The specific gravities are reduced to vacuum and 0° Cent. in terms of water at its greatest density.

No.	Sort of Glass.	Specific Gravity.	B	C	D	E	F	G	H	
1	Light Flint	3·2177	1·57816	1·57955	1·58369	1·58894	1·59355	1·60267	1·61073	not quite so colourless as Nos. 2, 3, and 4.
2		3·1530	1·56416	1·56558	1·56945	1·57449	1·57890	1·58750	1·59502	colourless.
3		3·5165	1·60510	1·60679	1·61147	1·61767	1·62316	1·63399	1·64421	colourless.
4	to	3·6155	1·61470	1·61647	1·62138	1·62787	1·63355	1·64469	1·65491	colourless.
5		4·1408	1·68539	1·68770	1·69388	1·70207	1·71001	1·72501	1·73897	slightly greenish-yellow, closely resembling the colour of chlorine.
6		4·4374	1·70422	1·70664	1·71346	1·72241	1·73068	1·74689	1·76166	decidedly greenish-yellow; chlorine colour.
7	Extra dense Flint	4·9566	1·77362	1·77669	1·78537	1·79697	1·80771	1·82917	1·84913	very strongly greenish-yellow, about $1\frac{1}{2}$ times the intensity of No. 6, but nevertheless apparently just as transparent, if not more so, and, considering the much greater dispersive power, a better prism.
8	Crown (Feil)	2·4688	1·50522	1·50621	1·50867	1·51175	1·51451	1·51964	1·52389	colourless.
9	Crown (Chance)	2·4728	1·51151	1·51254	1·51488	1·51804	1·52073	1·52587	1·53023	colourless.
10	Flint (Chance)	3·6619	1·61706	1·61868	1·62365	1·63012	1·63594	1·64775	1·65828	colourless.

Corrections to *Monthly Notices*, present volume, p. 438.

In the column $\Delta\delta$ the minus sign applies to all the quantities.

In the column "Observer" *for* T. G. L. *read* J. G. L.

After the observations *insert*: The observers were J. Gerh. Lohse and Ralph Copeland.